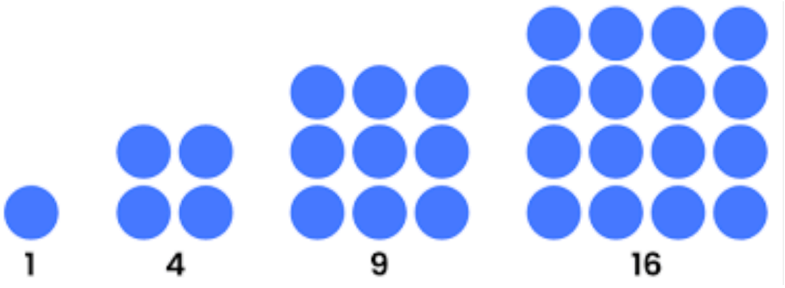
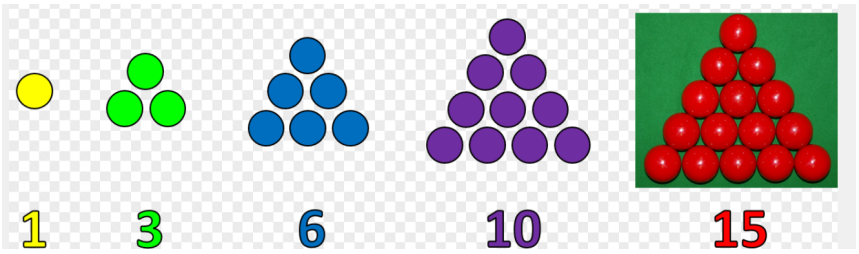
Square numbers



Triangular numbers



Square Triangular Numbers

1 appears on both lists so is a **square triangular number.**

0 could also be described as a square number (0x0) and is technically the first triangular number so 0 is also a **square triangular number.**

Can you find any more **square triangular numbers** (numbers that appear on both lists)?

Facilitator notes

- Should be easy to find 36
- Could use excel to find more: 0, 1, 36, 1225, 41616, 1413721, 48024900, 1631432881
- Once you have found a few numbers you can find the prime factorisation to see a pattern

$1 = (1 \times 1)^2$

$36 = (2 \times 3)^2$

$1225 = (5 \times 7)^2$

$41616 = (12 \times 17)^2$ (12 not prime, need to choose carefully to see pattern)

$1413721 = (29 \times 41)^2$

Next one will be $((29 + 41) \times (29 + 41 + 29))^2 = (70 \times 99)^2 = 48024900$

In general if you have a solution $(a \times b)^2$ the next solution is $((a + b) \times (2a + b))^2$

There is also a generating sequence

$S_{n+2} = 34 \times S_{n+1} - S_n + 2$

Which gives all the numbers if you start with $S_1 = 0$ and $S_2 = 1$

Extension

Trying to solve $\frac{1}{2}n(n+1) = k^2$ for some n and k

$n(n+1) = 2k^2$

$n^2 + n = 2k^2$

$4n^2 + 4n = 8k^2$

$4n^2 + 4n + 1 = 8k^2 + 1$

$(2n+1)^2 = 8k^2 + 1$

$(2n+1)^2 - 8k^2 = 1$

This is equivalent to

$y^2 - 2x^2 = 1$

Where $y = 2n + 1$ and $x = 2k$

This is a Pell's equation (which was first worked on by Brahmagupta a century earlier) and has many ways to solve for integer values of x and y

One way is to find **a** solution that works and then use this identity with your x and y to get another solution

$(x^2 - 2y^2)^2 = (x^2 + 2y^2)^2 - 8x^2y^2$

E.g. x = 3 and 2 would work because

$y^2 - 2x^2 = 1$

$3^2 - 2 \times 2^2 = 9 - 8 = 1$

So then

$(3^2 - 2\times 2^2)^2 = (3^2 + 2\times 2^2)^2 - 8\times 3^2 \times 2^2$

$1 = 17^2 - 2 \times 12^2$

So $x = 17$ and $y = 12$ which gives $17 = 2n + 1$ and $12 = 2k$ so $n = 8$ and $k = 6$.

$6^2 = 36$ and $\frac{1}{2} \times 8 \times (8+1) = 36$