Schur Numbers

We are counting in whole numbers always starting at the number 1. 1, 2, 3, …..

Schur numbers tell you highest number that you can count to using k different colours before you're forced to have an all same-coloured solution to a + b = c.

You're trying to **avoid** this:

Two numbers of the same colour **add up to** another number **of the same colour**.

In other words:

- If you have **red 2** and **red 3**,
  then **red 5** would break the rule (because 2 + 3 = 5, and they're all red).

- If you have **blue 2**,
  then **blue 4** would break the rule (because 2 + 2 = 4 and they're all blue).

So, you want to colour the numbers so this never happens!

Example

- Let's say k = 1. For 1 colour, let's say red, we can only count up to the number 1.

1

The reason we can't write a red 2

1 2

is that **red 1 + red 1 = red 2** and we are not allowed to have two numbers of the same colour that add up to a number of the same colour.

So if we have only 1 colour, we can only count up to the number 1. S(1) = 1

- For k = 2. For 2 colours, let's say red and blue:

1 2

Now we have to decide what colour to colour the number 3. We can choose either colour we like, since the only way to make 3 is 1 + 2. 1 and 2 are already different colours, so we are guaranteed that a = 1, b = 2 and c = 3 won't **all** be the same colour.

If we choose red for 3, can you find a colour to colour the number 4?

1 2 3

What if we choose blue for 3, can you find a colour to colour the number 4?

1 2 3

With 2 colours, what is the highest number you can count up to without breaking the rule?

S(2) = ?

- For k = 3. For 3 colours, red, blue and green

Is this a valid colouring?

1 2 3 4 5 6 7 8

Is this a valid colouring?

1 2 3 4 5 6 7 8 9

Can you find the highest number you can count to using 3 colours?

S(3) = ?

Can you find S(4) ?

A computer was used to find S(5). How many bytes of data were needed to find S(5)?

Check out this numberphile video on Schur Numbers:
https://youtu.be/57V8Ud7PL8k