Three indistinguishable dice problem – Matt Parker

Task 1

There are 6x6x6 =216 different ways to throw three dice.

Here are the number of ways to make each sum e.g. 3 - 1,1,1 but 4 - 1,1,2 or 1,2,1 or 2,1,1.

Notice that the numbers of ways follow the triangular numbers up to number 8!

Sum	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Number of ways	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1

To map this for 1 dice we need to think about the probabilities. We need ⅙ chance for each outcome - which is equivalent to 36/216. For example the number of ways to make 4,5, or 10 is 3+6+27 = 36 so that could be equivalent to throwing one of the numbers on a one sided dice. We could combine sums from above as follows:

Sum	1	2	3	4	5	6
Number of ways	4,5,10	3,6,9	7,8	11,16,17	12,15,18	13,14

This would work, you would just need to look up in this table what your score on three dice corresponds to. But … we can rearrange these numbers to make it much nicer:

Sum	1	2	3	4	5	6
Number of ways	7,13	8,14	3,9,15	4,10,16	5,11,17	6,12,18

Now all you have to do is roll the three dice and work out the sum mod 6 and that is your one dice roll! If you are not sure what mod 6 is, it is equivalent to subtracting 6 until you get a number between 0 and 5 with 0 being the same as 6.

Task 2

How about for 2 dice?

Sum	2	3	4	5	6	7	8	9	10	11	12
Number of ways	1	2	3	4	5	6	5	4	3	2	1
Probability out of 216	6	12	18	24	30	36	30	24	18	12	6

It looks like we can't do the same trick as before as for example we can't make any any of the previous frequencies add up to 12. So we need a solution that doesn't involve the sum, or doesn't just involve the sum.

See Matt Parker's great video for several solutions https://www.youtube.com/watch?v=hBBftD7gq7Y

Here is a great succinct one:

@standupmaths
3 dice in ascending order a,b,c with sum s:
all same->6,6
two same->a,c
b-a=1->c,a
c-b=1->s-9,s-8 (-1,0->1,1)
else->s-7,s-7