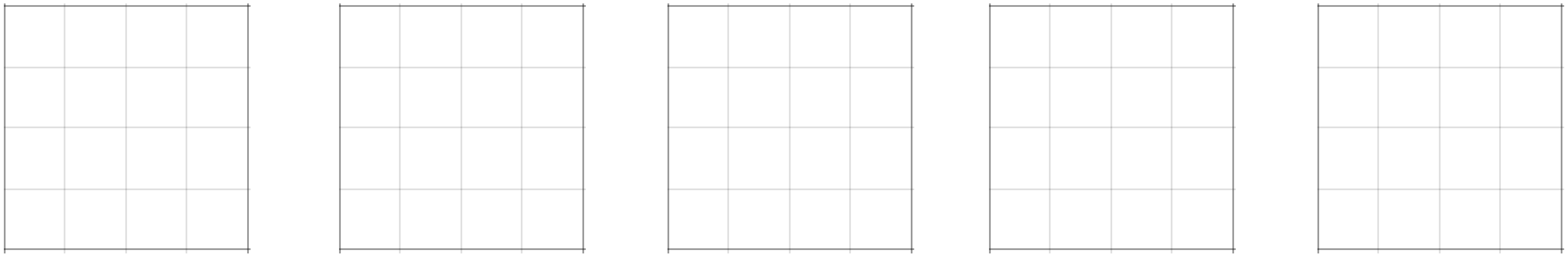
1. I select 11 positive whole numbers at random. Prove that two of them must have the same last digit.

2. There are 6 people at a party. Prove that 2 of them must have the same number of friends at the party. (Friendship is mutual, so if A is a friend of B, then B is a friend of A).

No three – in – a – line

3a. Take a 5×5 grid. Colour in as many dots (corners) as you can without making three dots that all lie on the same line.



3b. Find the maximum number of dots that can be coloured and prove that this is the maximum.

3c. Can you extend your result to a 5×5 grid? An $n \times n$ grid?

Don’t make a rectangle.

4a. Each of these dots is to be coloured red or blue. Can you find a colouring so that **no** rectangle is made with all 4 corners the same colour? If so, show it. If not prove that it is impossible.

4b. Each of these dots is to be coloured red, yellow or blue. Can you find a colouring so that **no** rectangle is made with all 4 corners the same colour? If so, show it. If not prove that it is impossible.