Hexadecimals and Binary

Adapted from: https://betterexplained.com/articles/numbers-and-bases/

Number Systems and Bases

Unary Numbers

Way back in the day, we didn't have base systems! When you wanted to count one, you'd write:

l

When you wanted 5, you'd write

lllll

And clearly, 1 + 5 = 6

l + lllll = llllll

This is the simplest way of counting.

Enter the Romans

Roman numerals decided they could do better. For five, they used V to represent lllll and get something like

l + V = Vl

And of course, there are many more symbols (L, C, M, etc.) they used.

Give each number a name

Another breakthrough was realizing that **each number** can be its own distinct concept. Rather than represent three as a series of ones, give it its own symbol: "3". Do this from one to nine, and you get the symbols:

1 2 3 4 5 6 7 8 9

Use position

Now clearly, you can't give **every** number its own symbol. There's simply too many.

But notice one insight about Roman numerals: they use the **position** of symbols to indicate meaning.

IV means "subtract 1 from 5"

and VI means "add 1 to 5".

In our number system, we use position in a similar way. We **always add** and never subtract. And each position is **10 times** more than the one before it.

100	10	1
	3	5
4	5	6

So 35 means $3 \times 10 + 5 \times 1$

And 456 means $4 \times 100 + 5 \times 10 + 6 \times 1$

This "positional decimal" setup is the Hindu-Arabic number system we use today. It is called **base 10**

Our choice of base 10

Why did we choose to multiply by 10 each time? Most likely because we have 10 fingers.

Considering other bases

We start a new column every ten. Our counting looks like this:

1, 2, 3, 4, 5, 6, 7, 8, 9 (uh oh, running out of digits), 10 (start a new column) and use 0 as a placeholder.

What if we started a new column at 60 when we counted, like we do for seconds and minutes?

1 second, 2, 3, 4, 5 …, 58, 59, 1:00 (60 seconds aka 1 minute. We've started a new column). **This is base 60.**

Base 16 - Hexadecimal numbers

If we want **base 16**, we could do something similar:

1

2

3

4

5

6

7

8

9

A (for 10)

B (for 11)

C (for 12)

D (for 13)

E (for 14)

F (for 15)

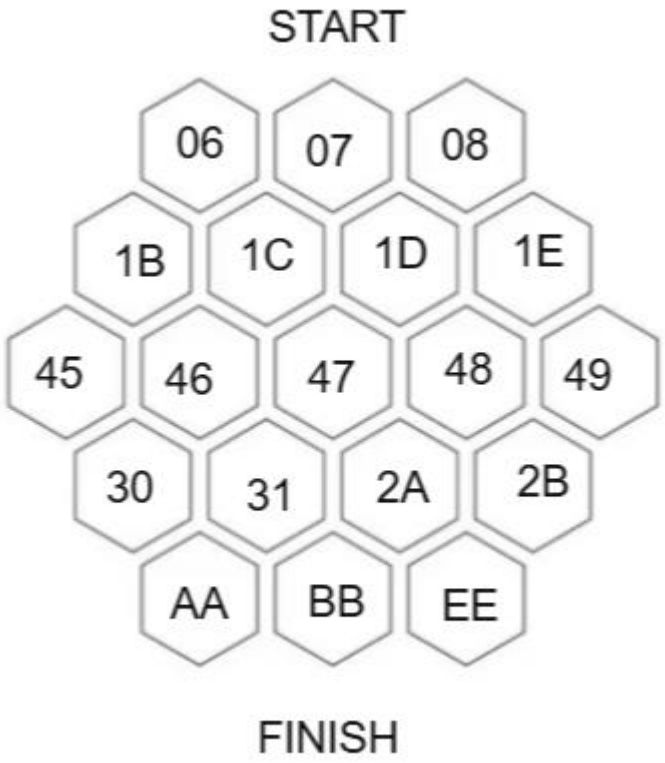
10 (for 16 – we've started a new column)

Examples of hexadecimal

Decimal	15	20	32	47	50	170	171	141
Hexadecimal	F	14	20	2F	32	AA	AB	8D

Activity 1

Colour in all the hexagons that are multiples of 7 when converted to decimal numbers.



Example:

16	1
1	C

1 x 16 +12=28
So can be coloured in

Activity 2

Fill in the table below. You will need to work out what the 3rd column represents in hexadecimal numbers

???	16	1

Decimal				17	88	740
Hexadecimal	11B	FFF	1000			

Activity 3

Find the sum of 3A5 + 2D1 by converting to decimal, doing the addition and converting back to hexadecimal.

Can you do it without converting to decimal?

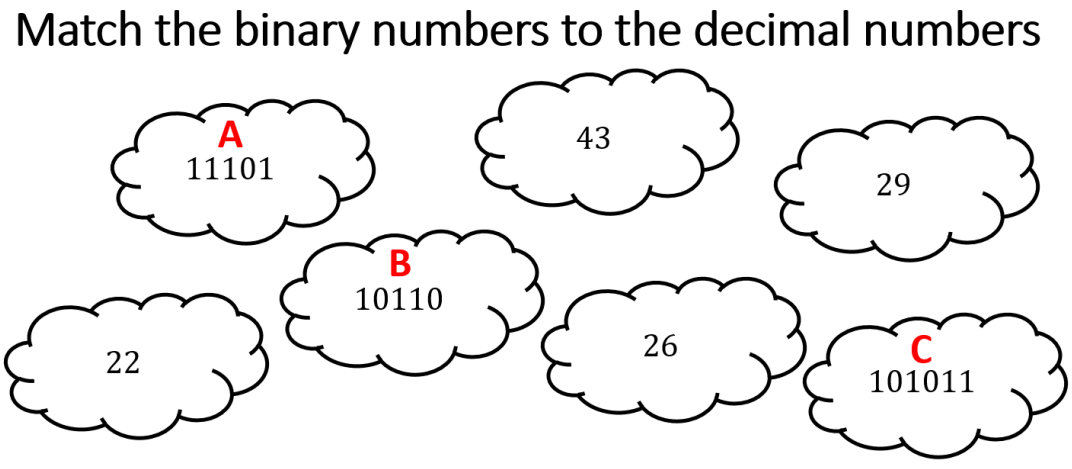
Binary

One of the most useful bases in the world is base 2, also known as binary.

Here are the numbers 0 to 15 in binary:

Dec	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

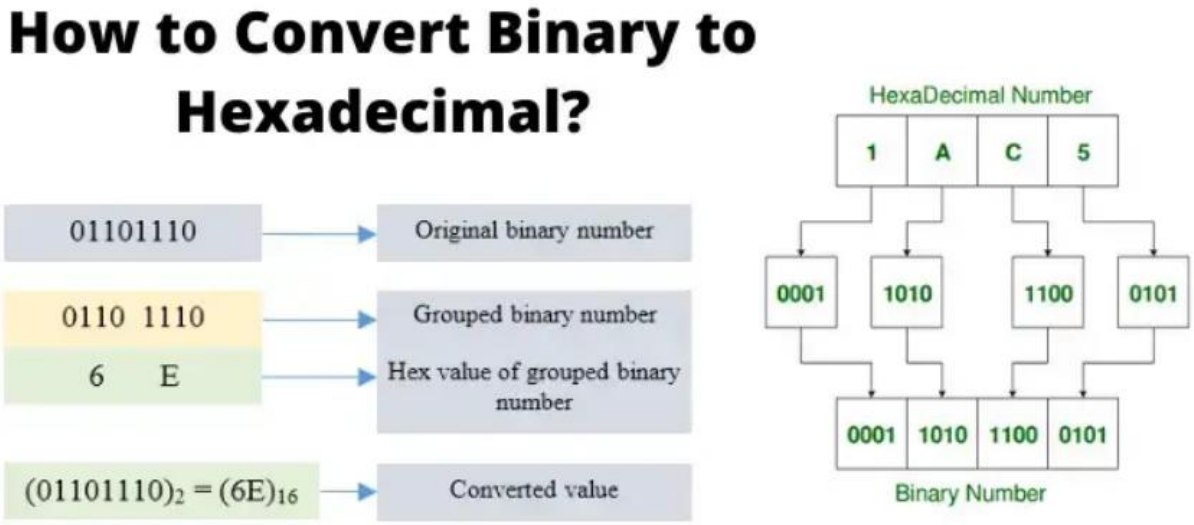
Activity 4



What's great about binary?

It's the simplest number system as it only uses two digits - 1 and 0. This means it is very easy to build in hardware. You just need things that can turn on or off (representing 1 and 0), so it is fundamental to all computers. For example, in a transistor, '0' means no electricity is flowing, whilst '1' means there is a flow of electricity.

How are binary and hexadecimals linked?



Because one block of four binary numbers represents one hexadecimal number it is easy for programmers to visualise numbers in hexadecimal. Programmers sometimes use words written in hexadecimal in their programs.

Activity 5

What word is this?

1101 1110 1010 1101 1011 1110 1110 1111

Activity 6

From https://nrich.maths.org/problems/base-puzzle

Find the missing number

10000, ? , 100, 31, 24, 22, 20, 17, 16, 15, 14, 13, 12, 11, 10

Hint at the bottom of this page

It is to do with the number 16