27 Card Trick

Understanding the trick

Take 27 cards from a deck

- Select a card and shuffle it back in.
- Deal out the cards face up in to 3 piles.
- Find your card and collect the piles back in. Place the card with your pile in the bottom pile, keep these cards face up but the remaining face down.
- Deal out the cards again, leave all cards face down apart from those that are face up.
- What positions are the identified cards in? Either describe or draw a diagram below.

- Find your chosen card. Collect the cards back in, keeping your chosen pile as the top pile, and collecting all the other cards in face down in the second and third pile
- Deal the cards out again in to 3 piles.
- Where are your chosen cards now? Either describe below or draw a diagram.

- Identify your card, collect the cards in again with your chosen card in the middle pile. When you deal out the cards your card should now be in 12th position.

Exploring the trick

- For the final stage, where would your card have been if you collected your pile in on the top, or the bottom, in comparison to the middle?

- What do you notice about those numbers?

The mechanics of the trick

- This trick works on working out how many 9s (3^2), 3s (3^1) and 1s (3^0) make the position number. We will be converting the position number in to a ternary number.
- 723 in base 10 = $7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$
- We need to convert between ternary numbers and base 10.
- Complete the table below

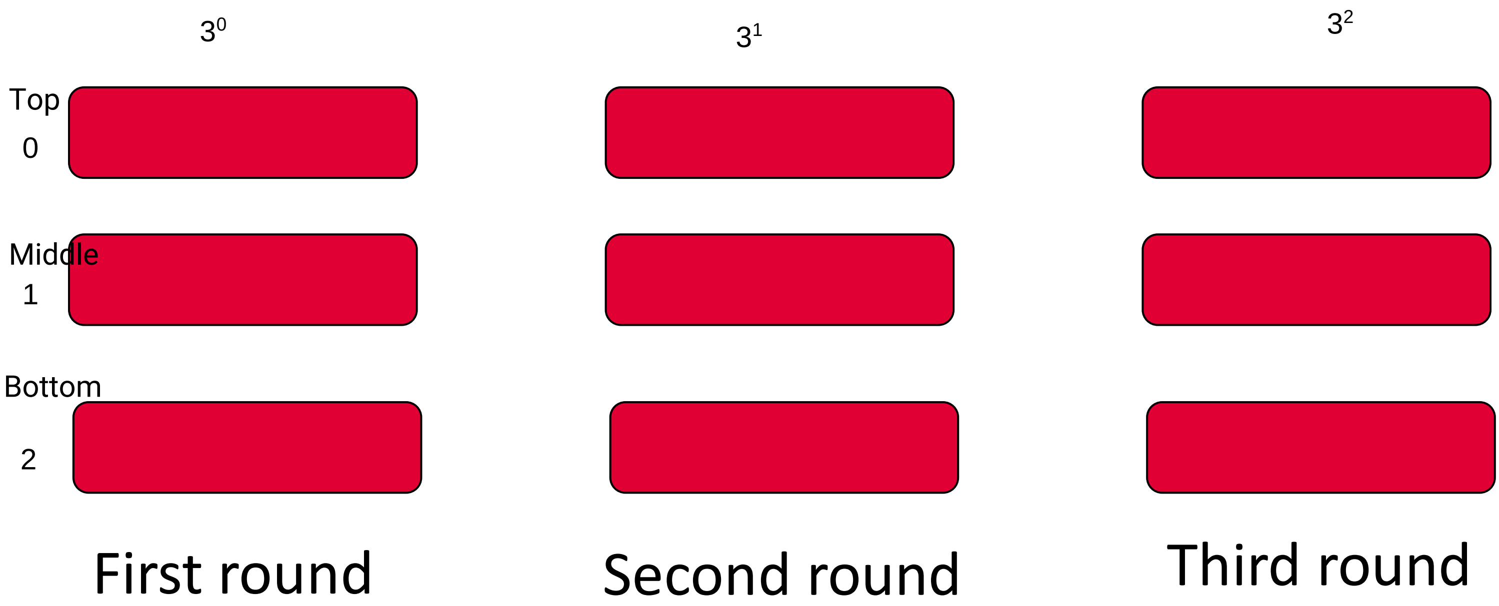
3^3	3^2	3^1	3^0	Number in base 10
2	1	1	0	2x27+1x9+1x3+0x1=66
	2	0	1	2x9+0x3+1x1 =19
	1	2	0	
1	1	0	2	
	2	2	1	

- Convert the following numbers to base 3:

1. 16 ________________
2. 20 ________________
3. 8 ________________
4. 9 ________________
5. 0 ________________

Performing the trick

1. Ask your partner to choose a number. Subtract from it and write it down ___________________
2. Turn your number in to ternary. ___________________
3. Reverse it ___________________
4. Carry out the trick, choosing the piles using this diagram – you may wish to annotate the diagram to track where you will be placing the piles.



Extension

- This trick can be used with any number that can be expressed as n^k where k and n are integers.
- For example you could use it with 49, 25, 1000 etc.

- Can you describe how you would put a card in the 16th place if you had 25 cards?

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- What might be the problem with using 1000 cards?

- Martin Gardener suggested the 10 billion card trick. You split 10 billion cards in to 10 piles and carry out the trick 10 times. This enables you to place a card anywhere you choose within the 10 billion deck!
  1. Estimate how long it would take you to carry out this trick
  2. Estimate how much space you would need to do this trick
  3. Carry out any other calculations you think might be interesting about this trick.