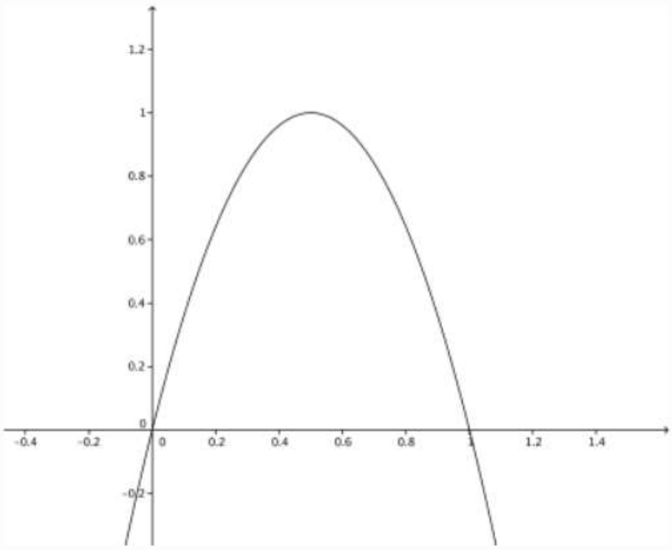
# Chaos theory - bouncing ball

Consider the quadratic equation:

$$y = 4x - 4x^2$$

It seems like a pretty ordinary equation, nothing much to get excited about. Here is its graph:




Use your calculator to plug in some numbers. Start with a value for x, say x=0.2, apply your equation and get y=0.64.

Now try again, this time using this new value for x: plug in x=0.64 and get y=0.9216.

Go again, with the new value x=0.9216 and get y=0.28901376.

Carry on repeating this process (called iterating) for as long as takes your fancy, and you'll get a sequence of numbers:

0.2, 0.64, 0.9216, 0.28901376, _________________, _________________, _________________

What do you think would have happened if you had started this process with a value of 0.2001 for x rather than 0.2? Those two numbers are very close, almost the same, so you would think they'd produce a fairly similar sequence of numbers...

_____,_____,__________,______________, _________________, _________________, _________________

Try this on a spreadsheet and graph your two sequences (against sequence number). What do you notice?

Then watch https://www.youtube.com/watch?v=6z4qRhpBIyA

Then try and recreate by adapting this trinket https://trinket.io/python/a900f88d52.

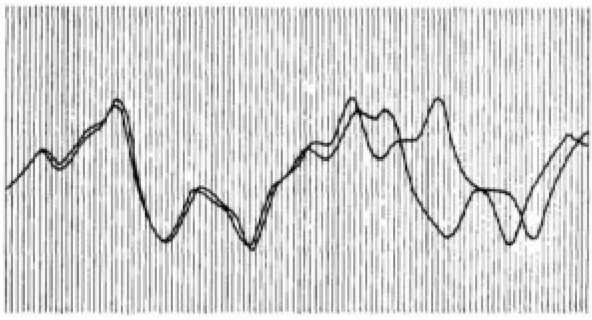
# Circa January 1961: Lorenz and the Butterfly Effect

To the average layperson, the concept of chaos brings to mind images of complete randomness. Yet to scientists, it denotes stochastic behavior occurring in a deterministic system: namely, systems that are so sensitive to measurement that their output appears random, even though there is an underlying order. This seemingly paradoxical viewpoint was born when a mathematician turned meteorologist named Edward Lorenz made a serendipitous discovery that subsequently spawned the modern field of chaos theory and changed forever the way we look at nonlinear systems like the weather.

Even as a boy, Lorenz was fascinated by the weather, monitoring the thermometer and recording highs and lows outside his parents' house in West Hartford, Connecticut. He was also interested in mathematics, often solving puzzles with his father. After graduating from Dartmouth College in 1938, Lorenz planned to go into math, but World War II intervened: he served as a weather forecaster in the Army Air Corps. Afterwards, he decided to stick with meteorology, making an early name for himself by publishing on such topics as the general circulation of the atmosphere.

But he was particularly intrigued by weather prediction, which was still largely intuitive guesswork, despite the assistance of scientific instrumentation. With the advent of computers, Lorenz saw the chance to combine mathematics and meteorology. He set out to construct a mathematical model of the weather using a set of differential equations representing changes in temperature, pressure, wind velocity, and the like. By the early 1960s, Lorenz had managed to create a skeleton of a weather system from a handful (12) of differential equations. He kept a continuous simulation running on an extremely primitive computer, which would produce a day's worth of virtual weather every minute. The system was quite successful at pro ducing data that resembled naturally occurring weather patterns nothing ever happened the same way twice, but there was clearly an underlying order.

One day in the winter of 1961, Lorenz wanted to examine one particular sequence at greater length, but he took a shortcut. Instead of starting the whole run over, he started midway through, typing the numbers straight from the earlier printout to give the machine its initial conditions. Then he walked down the hall for a cup of coffee, and when he returned an hour later, he found an unexpected result. Instead of exactly duplicating the earlier run, the new printout showed the virtual weather diverging so rapidly from the previous pattern that, within just a few virtual "months", all resemblance between the two had disappeared.



At first Lorenz assumed that a vacuum tube had gone bad in his computer, a Royal McBee, which was extremely slow and crude by today's standards. Much to his surprise, there had been no malfunction. The problem lay in the numbers he had typed. Six decimal places were stored in the computer's memory: .506127. To save space on the printout, only three appeared: .506. Lorenz had entered the shorter, rounded-off numbers assuming that the differenceone part in a thousandwas inconsequential.

It seemed a reasonable assumption. Scientists are often taught that small initial perturbations lead to small changes in behavior in any given physical system, and even today, temperature is not routinely measured within one part in a thousand. Lorenz's computer used a purely deterministic system of equations, so that given a particular starting point, the "weather" would unfold exactly the same way each time, while a slightly different starting point would cause the weather to unfold in a slightly different way. Lorenz figured a small numerical variation was similar to a small puff of wind, unlikely to significantly impact important, large-scale features of the weather. Yet in Lorenz's particular system of equations, such small errors proved catastrophic. Today, this phenomenon is known as sensitive dependence on initial conditions. Lorenz subsequently dubbed his discovery "the butterfly effect": the nonlinear equations that govern the weather have such an incredible sensitivity to initial conditions, that a butterfly flapping its wings in Brazil could set off a tornado in Texas. And he concluded that long-range weather forecasting was doomed.

In the past, such observed behaviour namely, random fluctuations coming from what should be a completely deterministic set of equations had been discarded as simply an error in calculation. Lorenz was the first to recognize this erratic behavior as something other than error; what he saw was undeniable order, born out of randomness. Not only was this the first clear demonstration of sensitive dependence on initial conditions, but Lorenz showed that this occurred in a simple but physically relevant model.

Lorenz then created a new system with three nonlinear differential equations, a reduced model of convection known as the "Lorenz Attractor." He hypothesized that the graph he created to model the motion would either reach equilibrium and stop, or create a loop that would eventually be reformed and retraced, indicating a repeating pattern. Instead, his map displayed an infinite complexity, always staying with certain bounds, but never repeating itself either. It traced a distinctive double-spiral shape, aptly resembling a butterfly with its two wings.

Since Lorenz's discovery, computer modeling has succeeded in changing the weather business from an art into a science, yet beyond two or three days, even the world's best forecasts are still speculative, and beyond a week, they are worthless. Such is the paradox that is chaos.