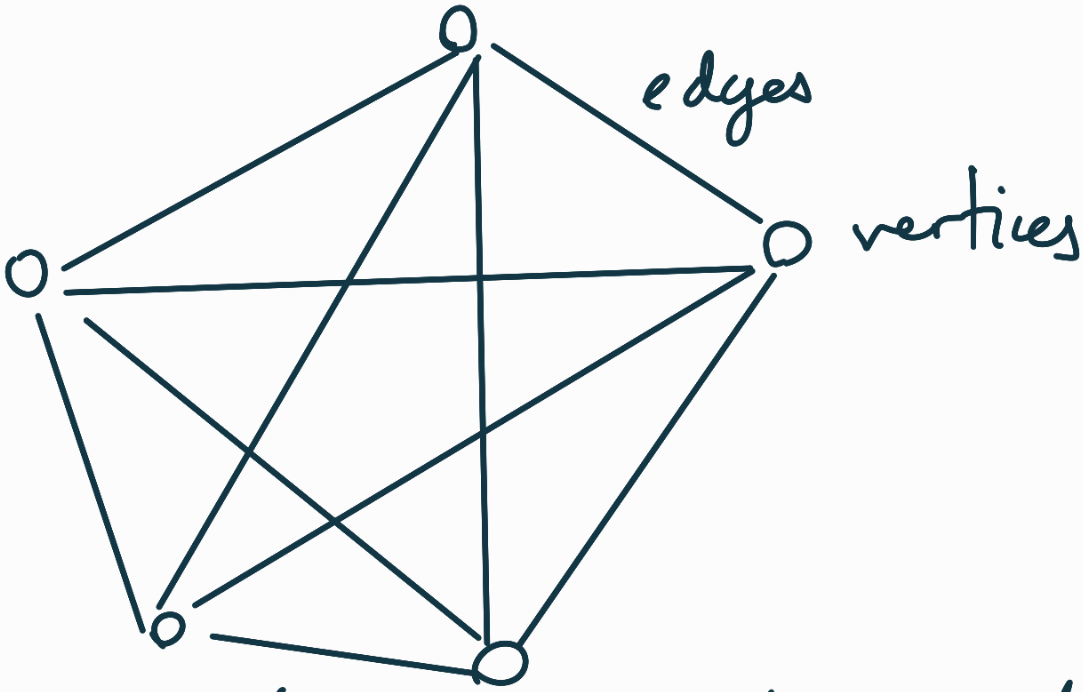
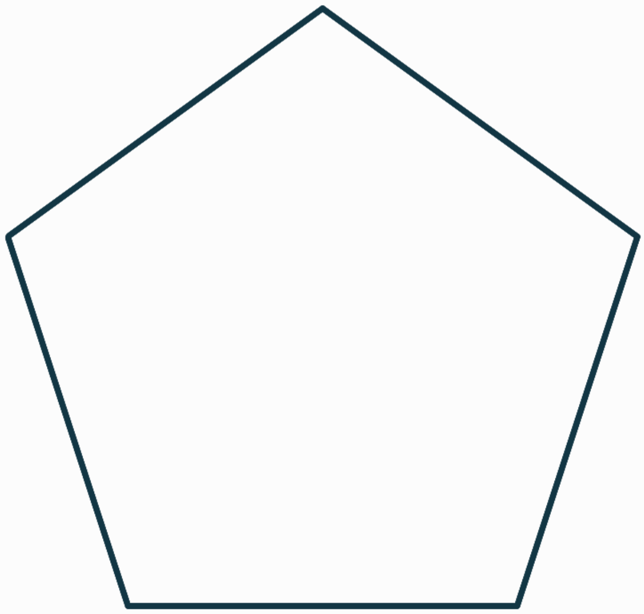
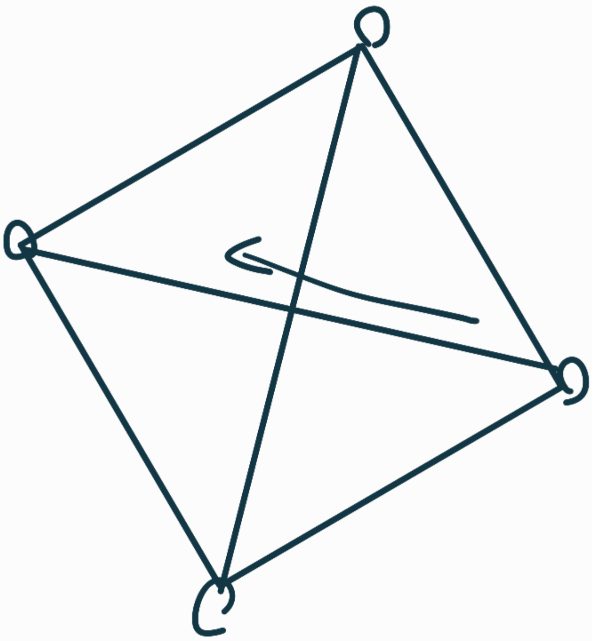
Graph Theory



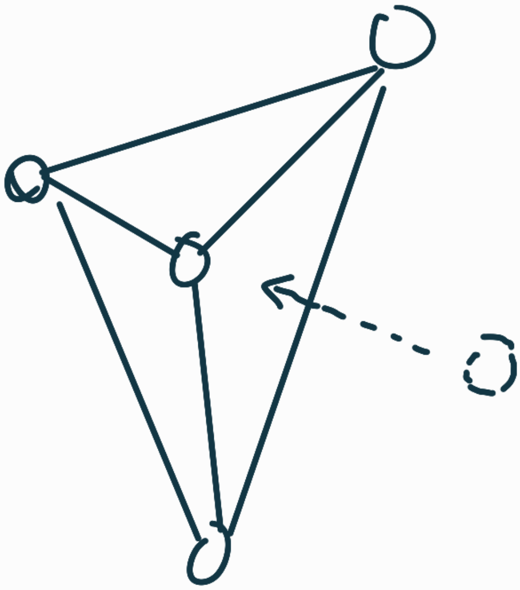
complete graph (all connected)

Planar graph (without edges crossing.





planar?



yes!

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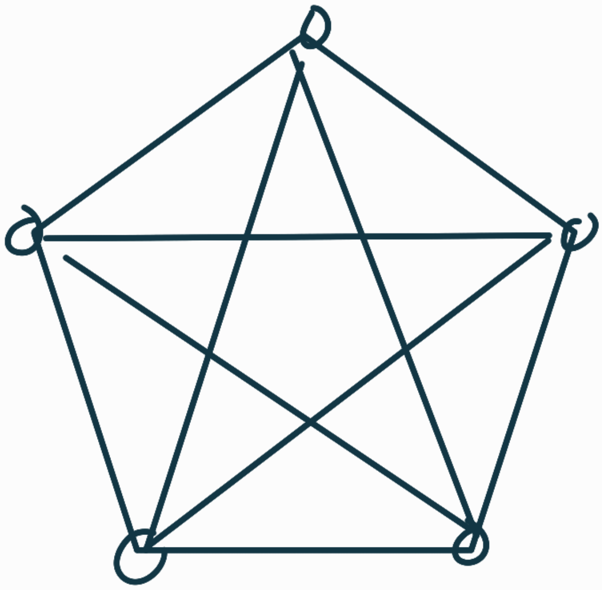
(1)

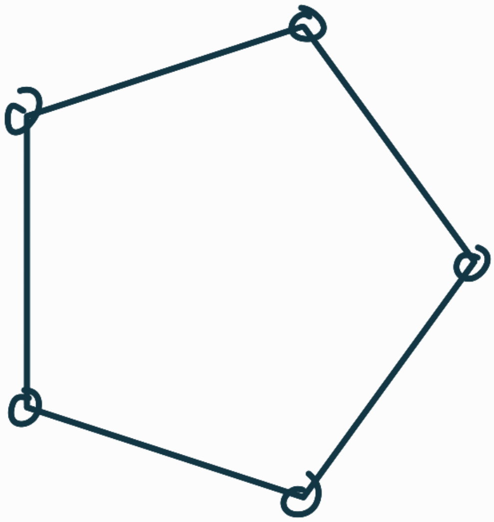
empty graph

(2)



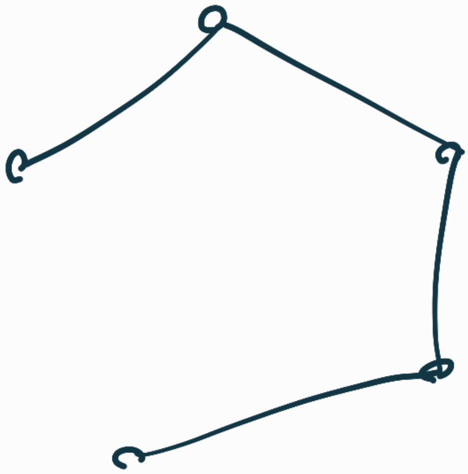
complete
K_n

(3)



cycle

(4)



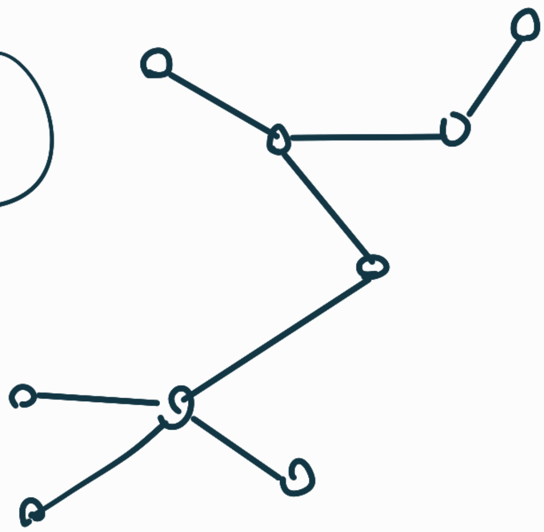
path graph.

path length 4.

or 

"connected means you can get there via any path"

(5)

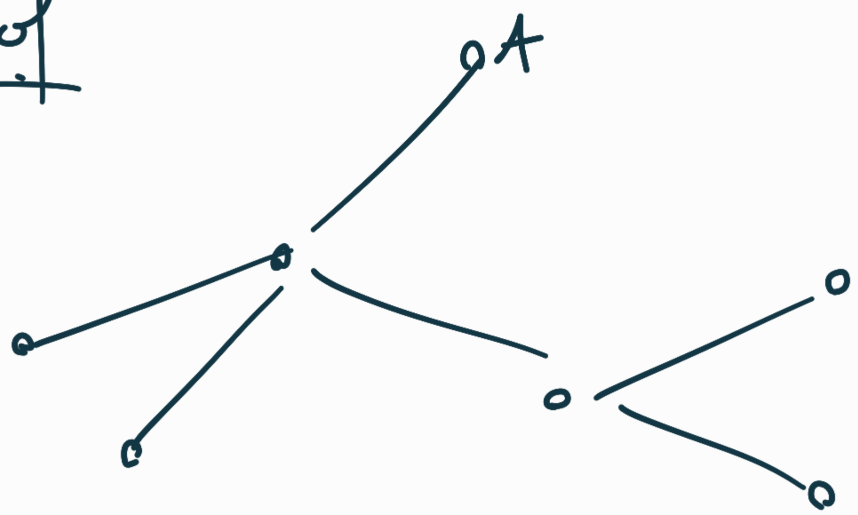


"tree graph"

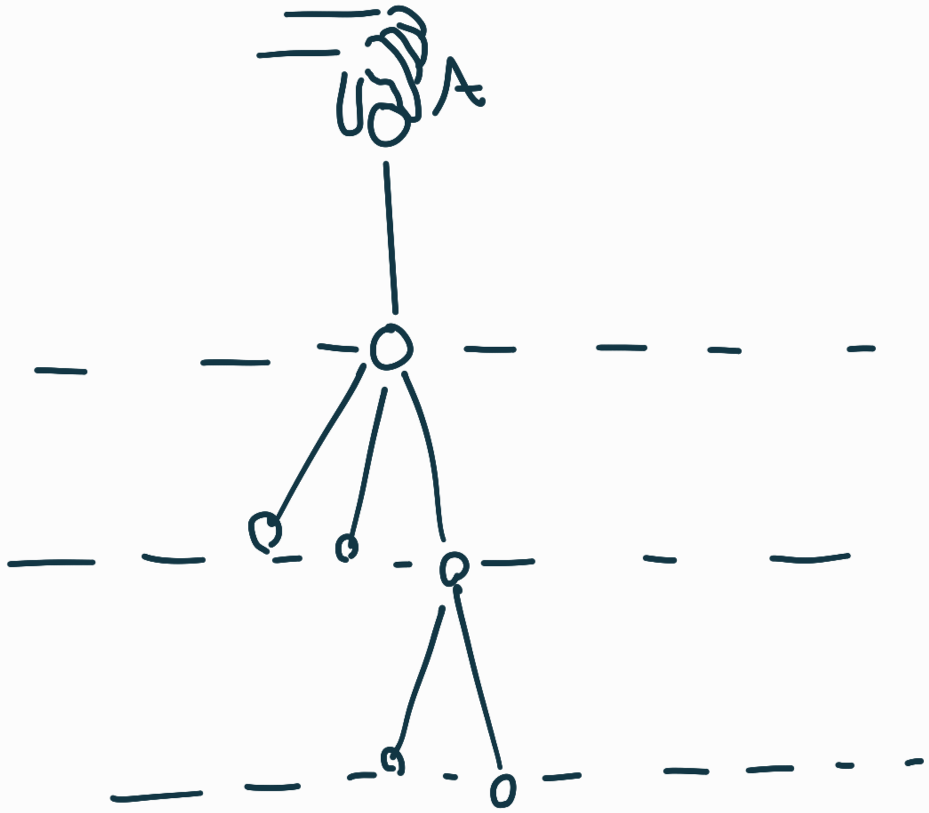
if a tree has n vertices

↪ it is going to have (n-1) edges

"proof"



I can pick A & "dangle under gravity"



at every level each vertex has 1 edge going up.
except the <u>top</u>! thus (n-1).

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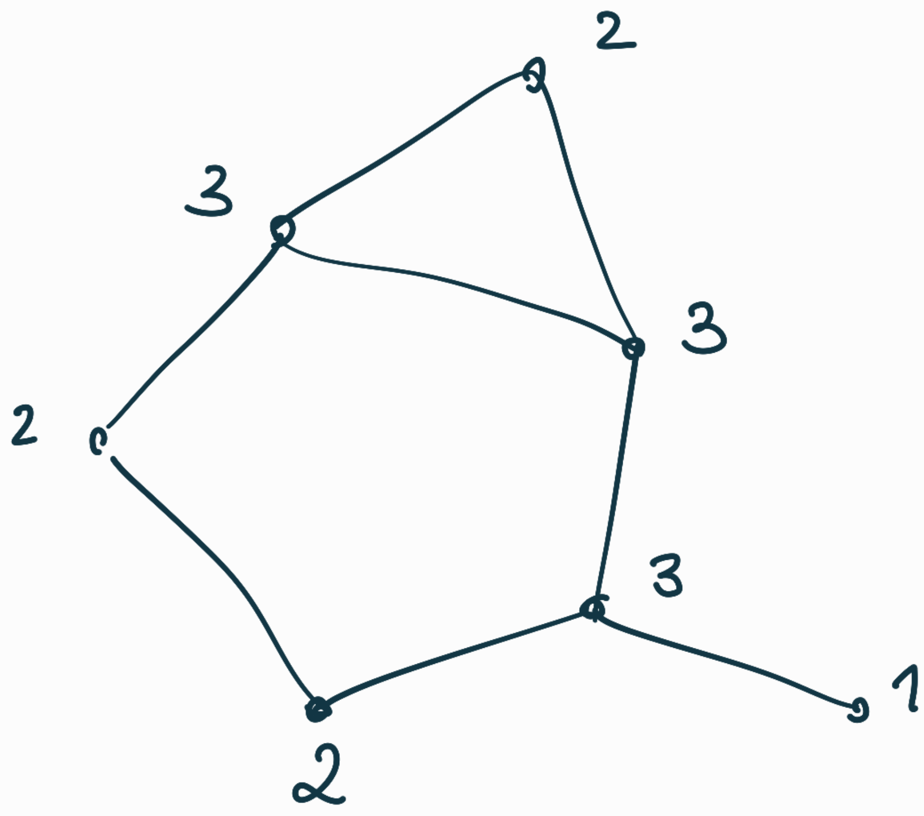
degree of a vertex

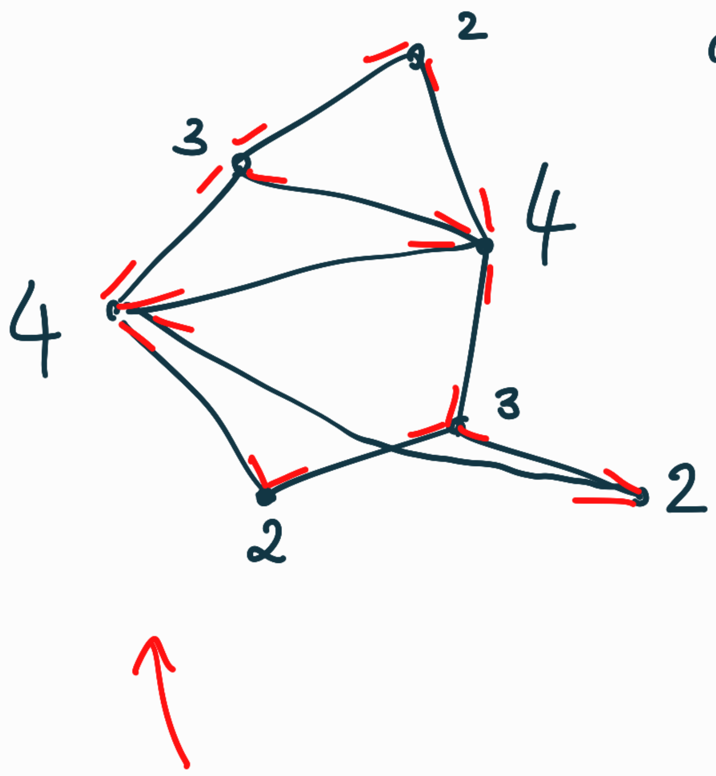
↪ number of edges "going out of it"



sum $2+2+3+1+3+3+2 = 16$

edges $= 8$

or



20 degrees

10 edges.

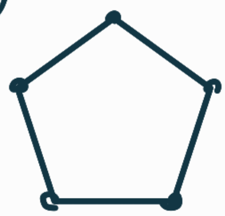
why?

each edge necessarily creates
2 degrees.

our first exercise.

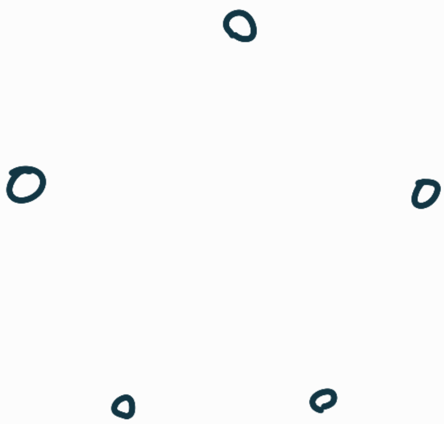
construct a graph in 5 vertices.
such that each has 2 degrees?

$\implies$ cycle



$\to$ has 3 degrees?

impossible?



↳ if sum of degrees = 2 × Σ edges.

↝ 3 each means 15 degrees.

↳ # edge $\notin \mathbb{Z}$

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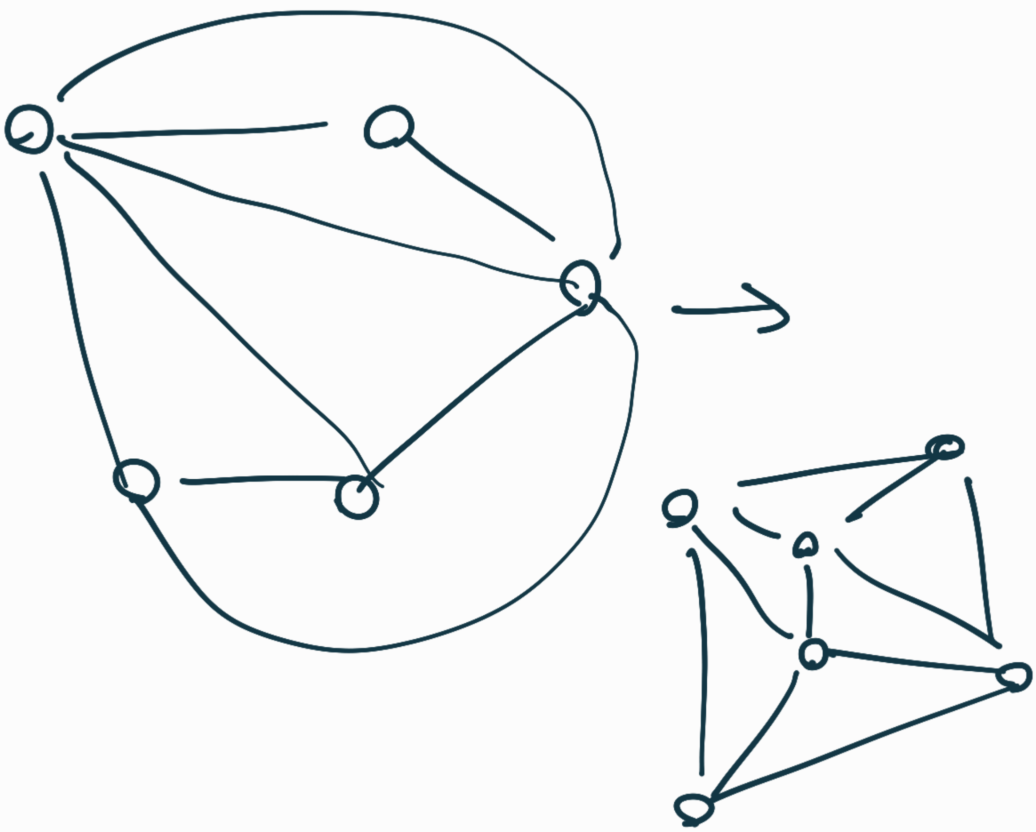
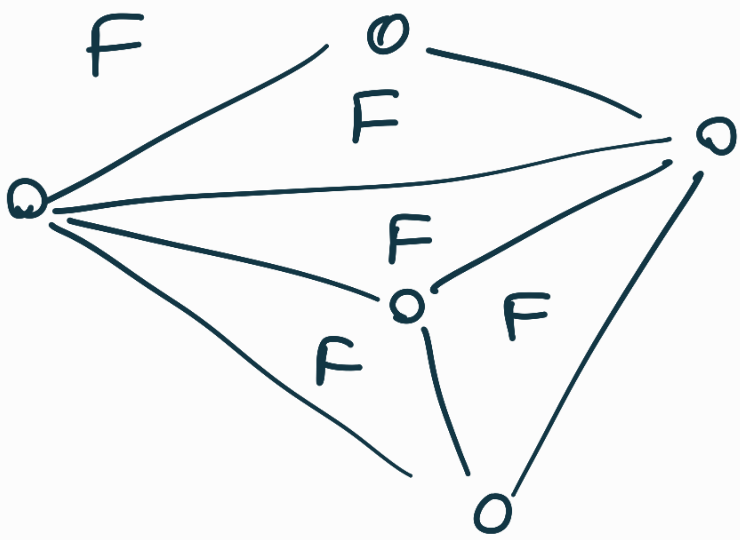
if your graph is planar you can always draw it with straight edges.

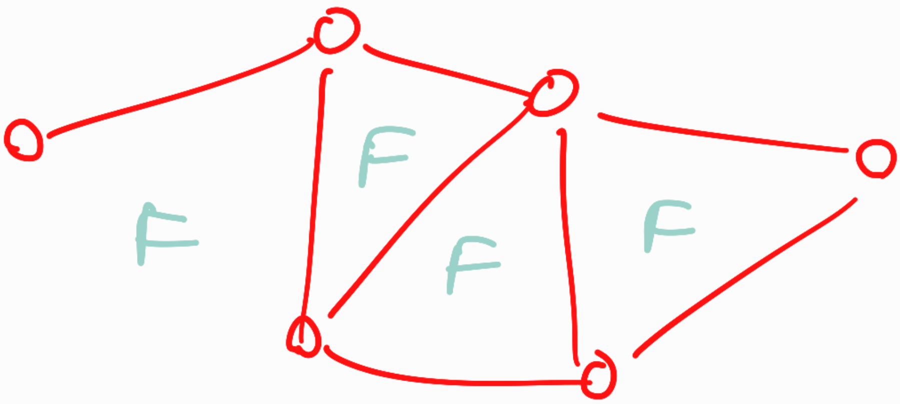


FACES



#F	#V	#E
5	5	8
4	6	8

other



$$\#V - \#E + \#F \; ?$$

$$5 - 8 + 5 = 2$$

$$6 - 8 + 4 = 2$$

⇒ as long as your graph is connected $\#V - \#E + \#F = 2$

EULER'S FORMULA

↳ 3d shapes?

↳ Cubes

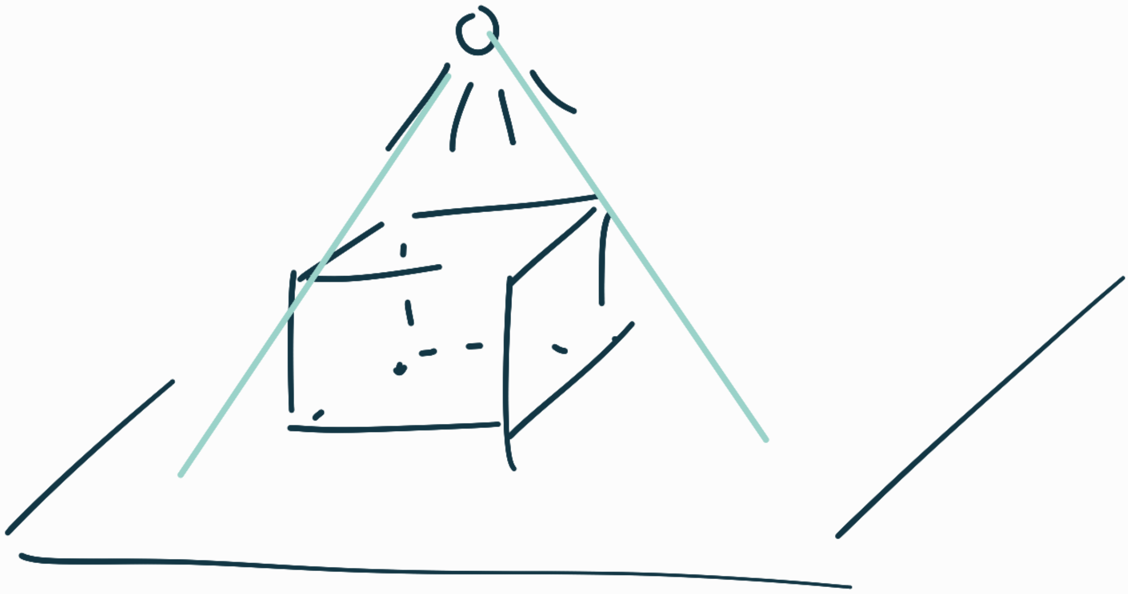
↳ Dodecahedron

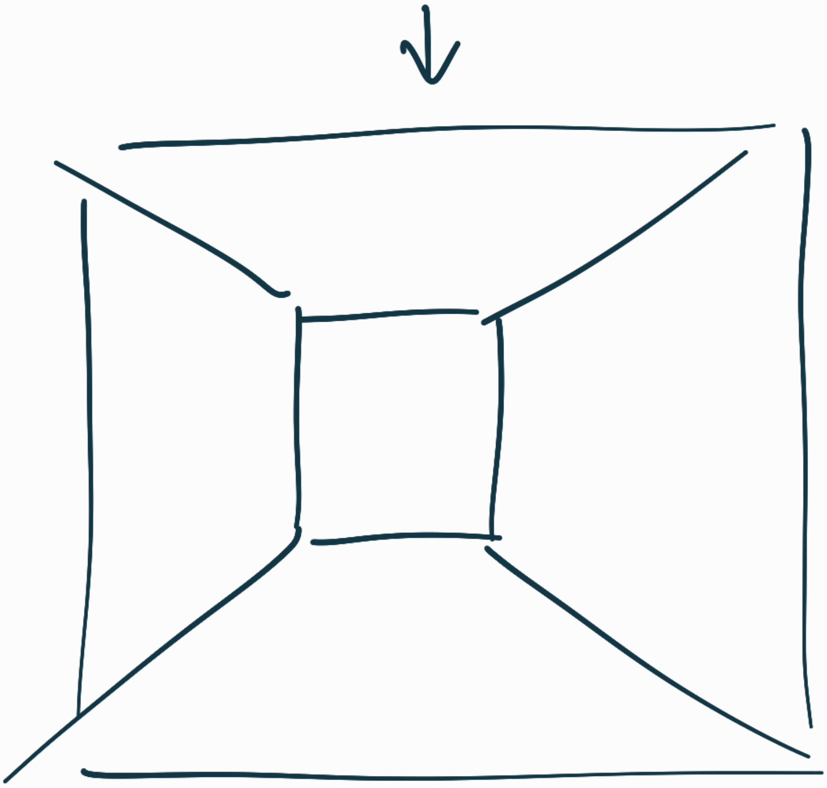
↳ Standard Football?

but not planar!

Projection

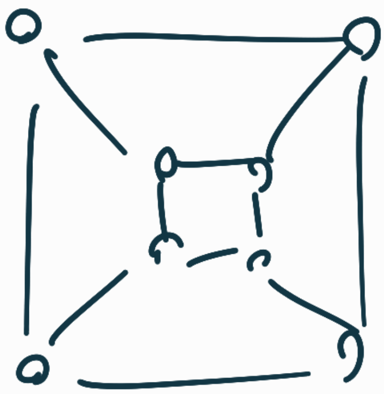
if you have a polyhedron you can project it onto a plane





and given that the outside is counted as a face

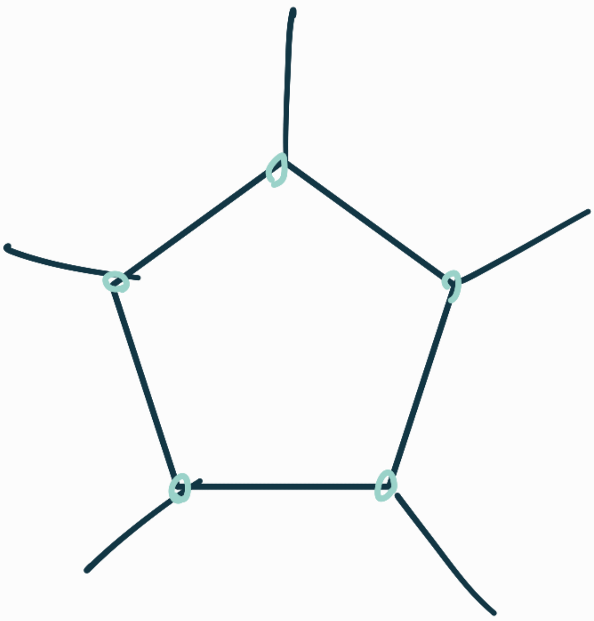
a cube can be represented as



↪ skip.

dodecahedron

one face



↪ $5 \times \#F = 2 \times \#E$

$3 \times \#V = 2 \times \#E$

↑ each V has 3 connections

or $\#F = \frac{2}{5} \# E$

$\#V = \frac{2}{3} \# E$

$$\left(\frac{2}{3}-1+\frac{2}{5}\right)\#E=2$$

$$\frac{1}{15}\#E=2$$

$$\#E=30$$

$$\#F=\frac{2}{5}\#E=\frac{2}{5}\times 30=12$$

$$\#V=\frac{2}{3}\#E=\frac{2}{3}\times 30=20$$

↳ properties of dodecahedron;

⟶ what about a football?