MATHS OF THE GAME – THE BACKGROUND

Designing this game was a highly mathematical experience!

This game is based on the game Ghost Blitz (Geistes Blitz), featured here on a board game website: https://www.board-game.co.uk/product/ghost-blitz-geistes-blitz/

Ghost Blitz works in the same way but uses five objects (a chair, a book etc) instead of shapes, and uses colour instead of style as the second property. Both Ghost Blitz and this game have been designed such that each object/shape matches up with an equal number of game cards in the pack.

But unlike Ghost Blitz we have designed this game so that each shape (with correct or incorrect styling) also appears the same number of times across the whole pack of game cards. In Ghost Blitz, some of the objects appear more times than others – for example the 'white bottle' appears seven times, but the grey book only appears twice.

Investigate with students

This is the mathematically sensible version of Ghost Blitz! There are 50 cards in our pack, a number that has been carefully chosen to allow the properties described above.

We'd love to hear about the maths your students find when they investigate this game. Get in touch with us at info@think-maths.co.uk. Alternatively, we have given some ideas on the pages below of the sort of questions you could get students thinking about.

Wrong/Correct shapes

In the questions below, we have used a few terms that need defining. The game cards feature both 'correct shapes' and 'wrong shapes'. 'Correct shapes' are shapes that appear in their correct styling, and 'wrong shapes' are shapes appearing with incorrect styling - for example the circle below is a 'wrong shape' because it is in the style of the triangle.

MATHS OF THE GAME – QUESTIONS

How to design?

1) How many different shapes (correct or wrong) are there that could be featured on the cards?

2) Imagine you have a type 1 (no correct shapes appear) card which ‘matches up’ with the square:

    a) Which type of shapes would not be able to appear on this card?

    b) How many different possible shapes could appear on this card?

    c) Design a set of cards (that are all type 1 and ‘match up’ with the square) where each of these possible shapes appears exactly once across the set. (Without looking at the set we designed!)

    d) How many different possible sets of cards could have been designed for part (c)?

How many cards?

1) How many different cards (of either type) could be made in this game? Assume the order of the shapes on the cards doesn’t matter.

2) We didn’t make them all! If we make all the possible cards this does not allow each of the shapes to appear the same number of times across the pack.

   We chose instead to make only 30 type 1 cards – across which each wrong shape appears an equal number of times. And we made only 20 type 2 cards.

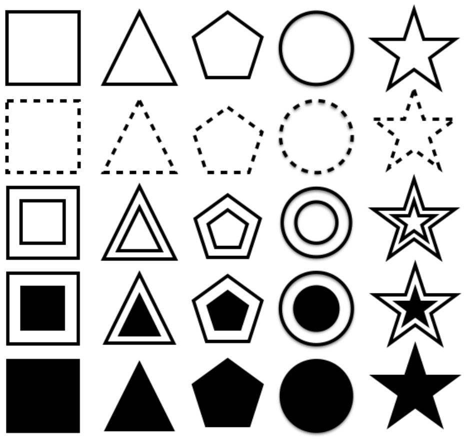
   Show that having 30 type 1 cards, and 20 type 2 cards allows every one of the shapes (correct or wrong) to appear the same number of times across the whole pack.

3) 50 is **NOT** the least number of cards required for each shape to appear the same number of times across the pack. How many would be the least (and with what split between type 1 and type 2)?

MATHS OF THE GAME – SOLUTIONS

How to design?

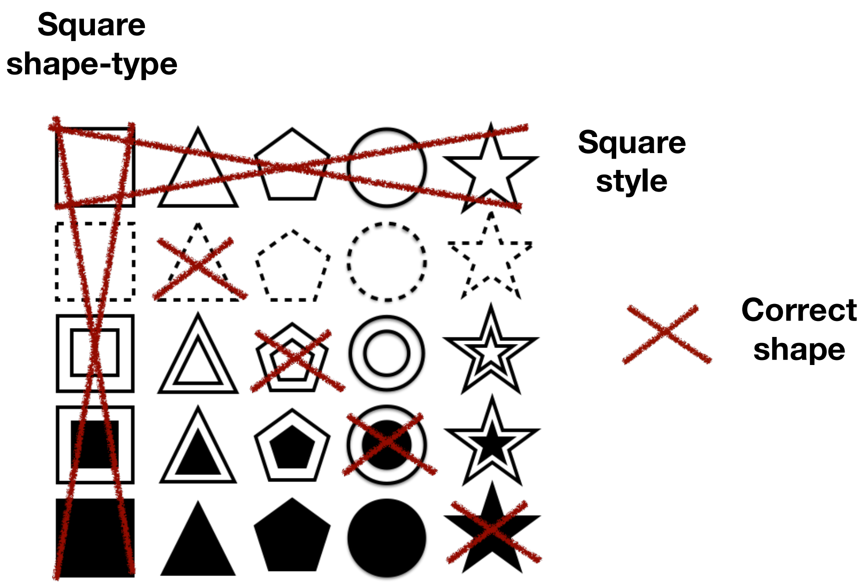
1) There are five styles and five shape-types, so there are 25 (5 × 5) possible shapes, featured below.



2)a) The five correct shapes do not appear on type 1 cards.

As the type 1 card matches up with the square, that means any shapes that share properties with the square cannot appear.

This removes squares with different styles, and different shapes in the style of the square (no-fill) – see the diagram below.



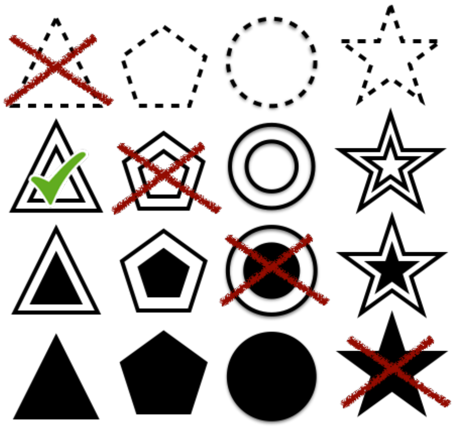
b) This leaves 12 possible shapes that could appear on the card.

c) There are 12 possible shapes to use once each, and two shapes per card, so the set must contain six cards.

One method for choosing the six pairings is outlined below.

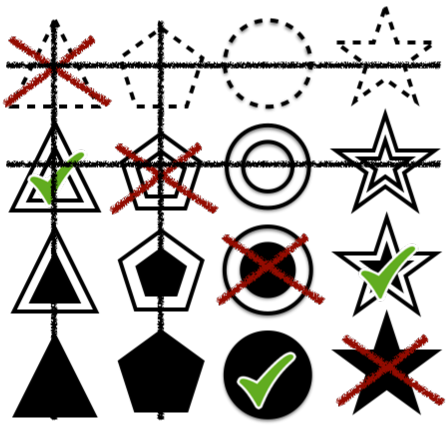
Removing shapes that share properties with the square leaves the grid below. Correct shapes are disallowed, but it's useful to leave them visible (but crossed out).

Choose a shape (green tick) to put on the first card.

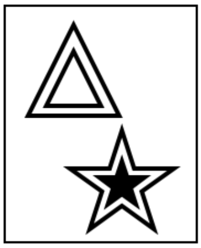


This chosen shape represents the triangle and the pentagon. The second shape on the card must take its properties from the two other shapes (in order to leave just one shape, the square, unrepresented).

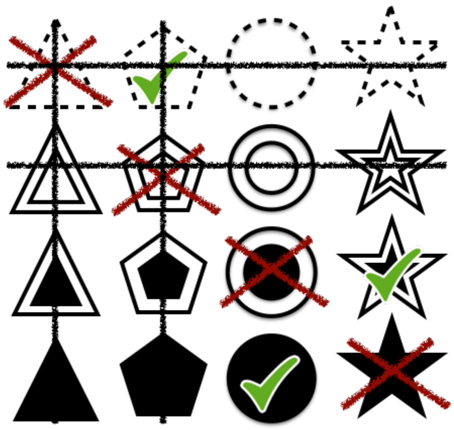
The second shape must either be a circle with star style, or a star with circle style. These two options are shown below with green ticks. (The shapes that have been ruled out share a property with the triangle or the pentagon, which have already been used).



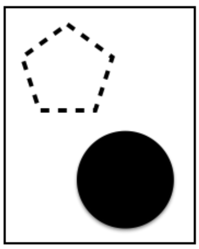
Then we choose one of these two to be the second shape on the card (as below).



The only other shape that has triangle and pentagon properties is the pentagon with triangle-style (chosen below). Choosing this for a card results in the same diagram and leaves the same two options for the second shape on that card.

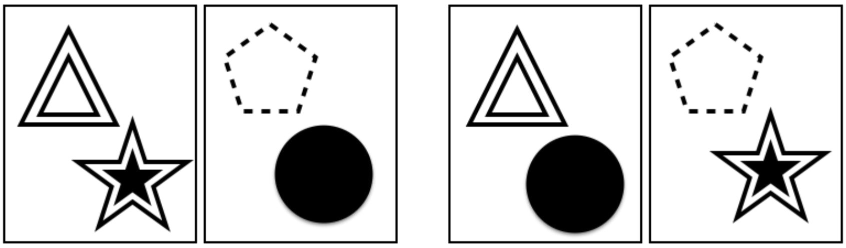


The circle/star shape which was not chosen to go on the previous card then goes on this card.



As we could use either of two options for the second shape on these two cards, this means there are two options for this pair of cards (below).

Option 1 Option 2

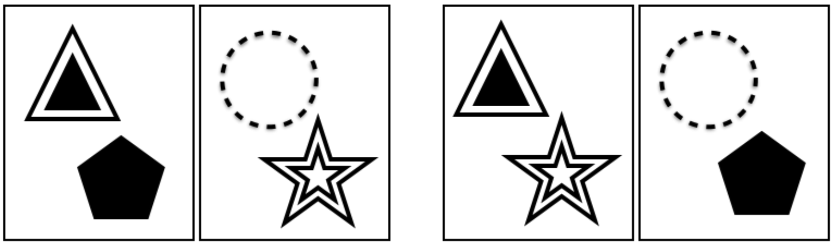


These are the: **triangle/pentagon - circle/star** cards.

(The first/second pair of words describe the properties of the first/second shape).

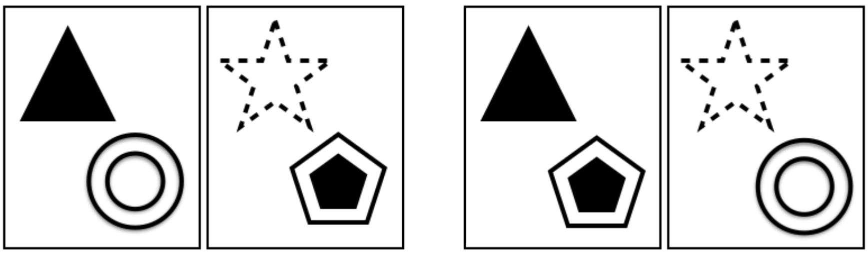
Using the same thinking, we see that there are two more sets of four cards that can be made, giving three sets in total. This makes sense; each set of cards represents two shape pairings, and there are 6 ($^4C_2$) pairs that can be made from four shapes. See the other two sets below.

Option 1 Option 2



The **triangle/circle - pentagon/star cards**

Option 1 Option 2



The **triangle/star - pentagon/circle cards**

A valid set of six cards is any made from a choice of either Option 1 or 2 for each of the three pairs of cards.

d) We've assumed that swapping the position of two shapes on a card does not count as making a new card.

With two options for each of the three pairs of cards (see above), the number of possible sets of six cards that could be created in part c) is therefore:

$$2^3 = 8$$

How many cards?

1) Find **how many possible type 1 cards there are**:

The four properties (two shapes, and two styles) on the card come from different shapes, so the number of possible cards is:

$5 \times 4 \times 3 \times 2 = 120$

But we don't care which order the shapes are on the card (and this total includes all the cards both ways round) so we halve this number to get **60 type 1 cards.**

Find **how many possible type 2 cards there are**:

Each of the five correct shapes can match up with any wrong shape that does not share its properties. There are $4 \times 3 = 12$ wrong shapes that do not share properties with any particular correct shape. So, there are:

$5 \times 12 =$ **60 type 2 cards.**

There are **120 possible cards** in this game.

2) Find **how many times the wrong shapes appear**:

There are 20 wrong shapes (five shape-types with four possible styles for each - the style cannot be correct). With two shapes per card, 30 type 1 cards would feature 60 shapes. We can choose the 30 type 1 cards so that each of the 20 wrong shapes appears three times each.

Each type 2 card contains one correct shape and one wrong shape. 20 type 2 cards could give us one more of each of the 20 wrong shapes.

So, there would be **four** of each wrong shape across the whole pack.

Find **how many times the correct shapes appear**:

As there are only five correct shapes, we can choose the 20 type 2 cards so that each of the five correct shapes appears **four** times each.

3) **25 cards** (**15 of type 1, and 10 of type 2**) is the least number of cards necessary.

The number of type 2 cards must be a multiple of five (so each correct shape appears the same number of times).

Let's imagine there are **five type 2** cards. This would give one of each correct shape. To also get one of each wrong shape we need 20 wrong shapes in total. We have one per card on the five type 2 cards, which leaves 15 shapes to appear over the type 1 cards. But the number of shapes in total on type 1 cards must be even as there are two per card.

Let's imagine there are **10 type 2** cards. This would give two of each correct shape. To get two of each wrong shape we need 40 wrong shapes in total. We have one per card on the ten type 2 cards, which leaves 30 shapes to appear over the type 1 cards. We can do this over **15 type 1** cards.

You may need to design the cards to convince yourself that it is possible to design the 15 type 1 cards and 10 type 2 cards such that each wrong shape appears exactly twice! Pick your own shapes and styles!