What's so special about the Mandelbrot Set?

What follows is tasks to help you understand and recreate the graphs in the amazing video by Ben Sparks: https://www.youtube.com/watch?v=FFftmWSzgmk

He is talking about the Mandelbrot set, which technically speaking is:

"The set of values of c in the complex plane for which the orbit of 0 under iteration of the quadratic map

$$z_{n+1} = z_n^2 + c$$

remains bounded"

Let's see what that means …

Task 1

$$z_{n+1} = z_n^2 + c$$

We are always going to be starting with $z_0 = 0$ but we are going to choose different values of c and see what happens to the sequence over time.

For example, let c=2

$$z_{n+1} = z_n^2 + 2$$

$$z_0 = 0$$

$$z_1 = z_0^2 + 2 = 0^2 + 2 = 2$$

$$z_2 = z_1^2 + 2 = 2^2 + 2 = 6$$

$$z_3 = z_2^2 + 2 = 6^2 + 2 = 38$$

$$z_4 = z_3^2 + 2 = 38^2 + 2 = 1446$$

The terms of the sequence will just get bigger and bigger.

Try these values of c and some of your own:

c=1, c=0.25, c=0.5, c=-2, c=-1, c=-4 ….

Let's colour in these values on a number line – black if the sequence seems to be bounded (stays between -1000 and 1000) and blue if not

-4	-3.5	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5

Task 2

Now let us move this into 3D.

Let $i = \sqrt{-1}$. Hence $i^2 = -1$

Now try the formula with c=1+i

$$z_{n+1} = z_n{}^2 + 1 + i$$

$$z_0 = 0$$

$$z_1 = z_0{}^2 + 1 + i = 0^2 + 1 + i = 1 + i$$

$$z_2 = z_1{}^2 + 1 + i = (1+i)^2 + 1 + i = 1 + 2i - 1 + 1 + i = 1 + 3i$$

$$z_3 = z_2{}^2 + 1 + i = (1 + 3i)^2 + 1 + i = 1 + 6i - 9 + 1 + i = -7 + 7i$$

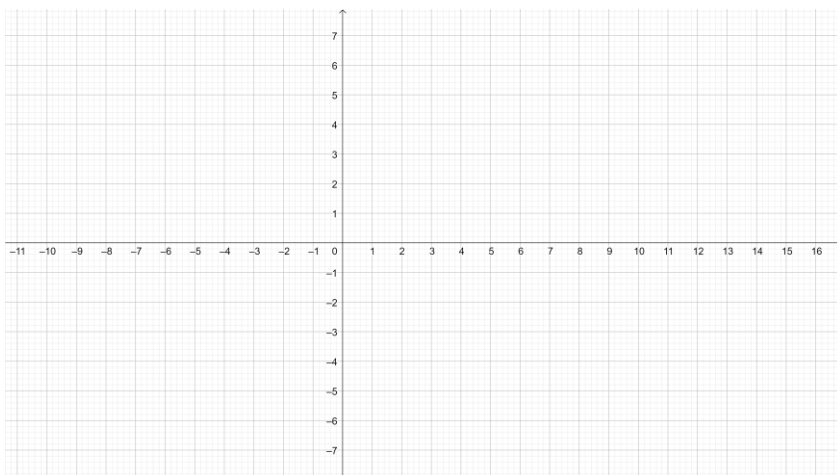$$z_4 = z_3{}^2 + 1 + i = (-7 + 7i)^2 + 1 + i = 49 - 98i - 49 + 1 + i = -97i + 1$$

On a calculator, the next iteration is -9407-193i. It seems like the numbers are not converging.

Try these values of c:

c = i

c= 0.5i (could do up to z_3 by hand, then use your calculator to write down z_5 and z_6)

If we coloured in c values like before, what would this look like on an argand diagram?

Task 3

Go to https://www.geogebra.org/classic or use a downloaded version on your computer.

Use the point tool [A•] to plot two points somewhere on the grid. Then use the arrow tool [↖] to move a point A to (0,0). Then move point B to (0,0.5). You might have to zoom in first using

[🔍] in the far right menu. It is important to plot the points away from the axes and then move them, otherwise they will be fixed to the axes and you won't be able to move them later.

To make this the same as the numberphile video, right click on point B and rename as C.

On the top right of the screen click on the three parallel lines:

[↩ ↪ 🔍 ≡]

And then select View, and tick the spreadsheet box.

You should now see a spreadsheet.

In cell A1 type:

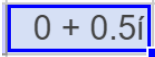=x[A] + i* y[A]

This will convert point A into a complex number
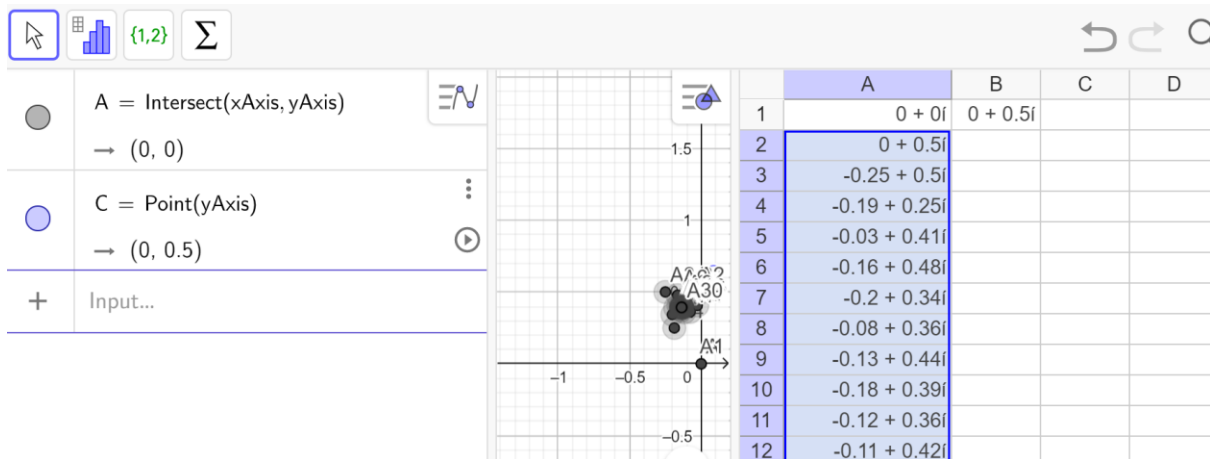
In cell B1 type

=x[C] + i* y[C]

In cell A2 type

=A1^2+B1

The dollar signs are to fix it so you are always adding what is in cell B1.

Click on cell A2 and drag the small square in the bottom right of the cell [0 + 0.5i] down to A30.

This should generate your sequence.

Your screen should look like this:

The numbers is column A are your sequence from earlier!!

Highlight cells A2 to A30 right click and untick the box Show Label
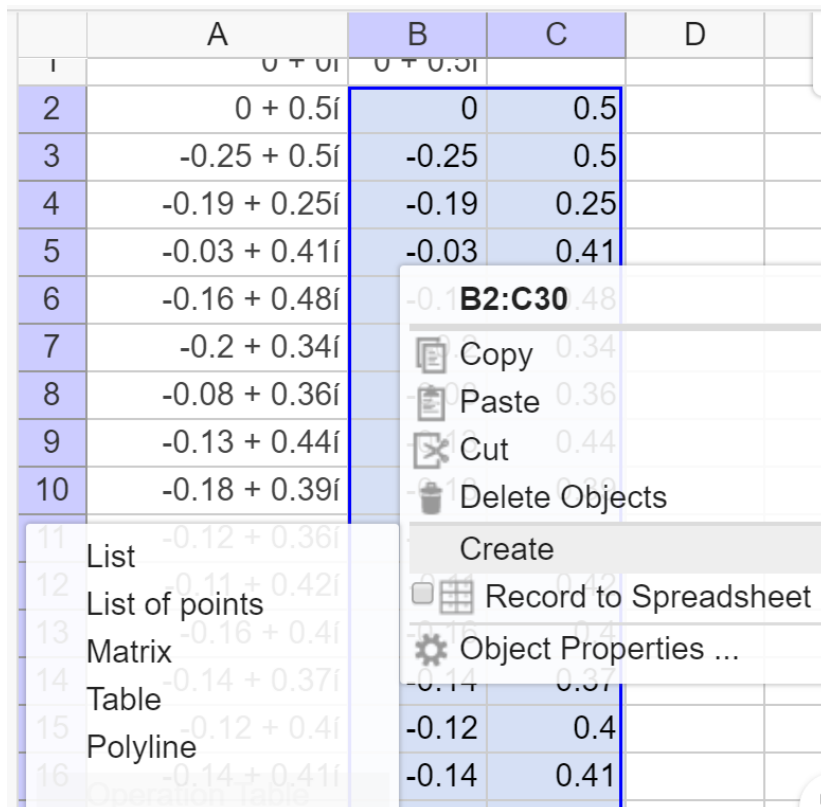
In cell B2 type

=real(A2)

In cell C2 type

=imaginary(A2)

You can now highlight these two cells and drag down again to row 30.

While you have cells in columns B and C all highlighted, rightclick and choose Create Polyline.
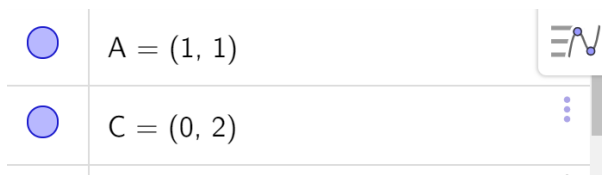
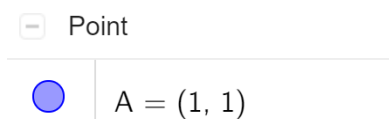You can now hide the spreadsheet by clicking on the three parallel lines:

And then select View, and tick the spreadsheet box.

Click on this symbol on the right of the inputs:

$A = (1, 1)$

$C = (0, 2)$

Then click on and choose Object Type. We are going to hide the label of all the points at the same time.

Click on the settings icon and then click on Point

Point

$A = (1, 1)$

In the box that shows up on the right, untick the box Show Label, then tick on the cross on the top left to close the settings.

You should now be seeing a visual representation of the sequence

$$z_{n+1} = z_n^2 + c$$

for c=0.5i

Now you can move B1 (which is the same as C) and see what sequences you get. Where there is convergence, that value of c is in the Mandelbrot set. Where there is no convergence it is not.

If you coloured in all the points C where there was convergence, what shape do you think it would make?