African Maths Initiative

P18 – Facilitator

Langford's Problem

Introduction

This puzzle can be done with pen and paper, or ideally it could be done with a pack of playing cards using the Aces, 2s, 3s, 4s and 5s. Students can then rearrange the cards to try and solve the three puzzles. The first puzzle is fairly easy, the second one is quite hard, and the third one is impossible! The proof that the third one is impossible is interesting because it uses colours – not all proofs involve algebra.

Solution

312132 (or the other way round – 231213)

41312432 (or the other way round – 23421314)

Adding in the 5s makes this impossible

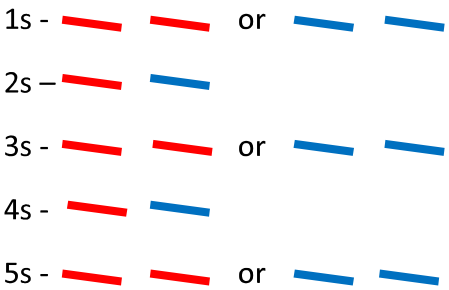
Imagine the ten numbers are written in this line:

The two 1s would have to be on the same colour lines to have one number between them.

The two 2s would have to be on different colour lines to have two numbers between them.

The two 3s would be on the same colour, 4s on different colours and 5s on the same colour.

So



There are five reds and five blues available in the line, but the best you can do with these constraints are 6 blues and 4 reds, or 6 reds and 4 blues. You cannot have five of each, so there is no way to fill in the numbers 1122334455.

Extension

Let n be the biggest number. So for n=3 and n=4 there is a solution, for n=5 there is no solution. For more information on which values of n in general have solutions see http://dialectrix.com/langford.html.