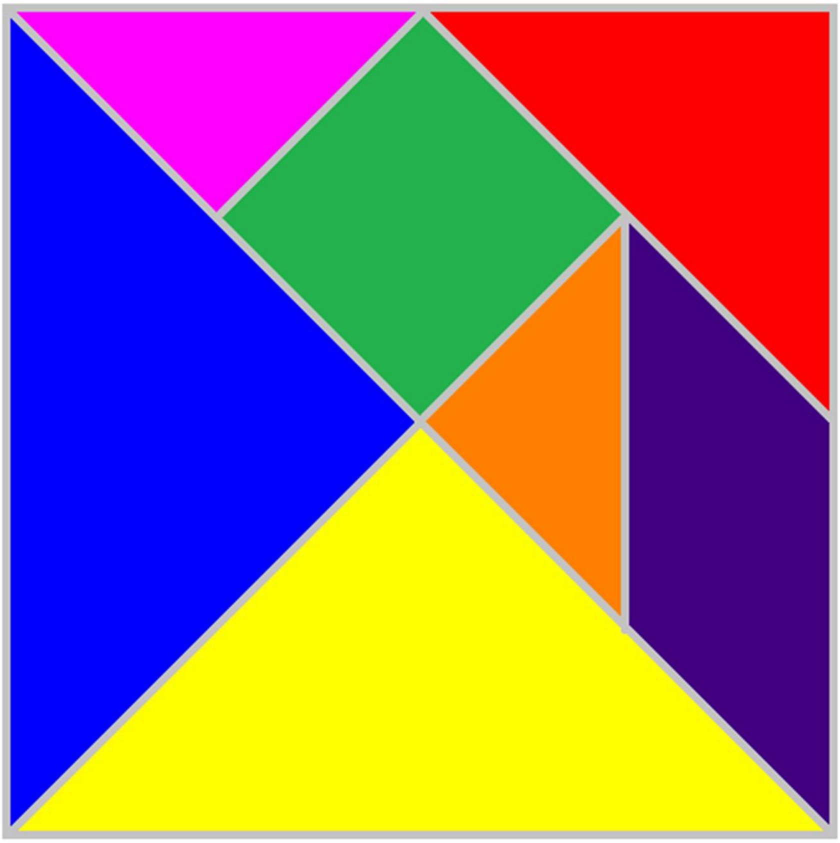
Tangrams

You will need pen, paper and either scissors or a ruler for this game

Create a tangram by tracing the image below onto a square of paper, or try to construct using paper folding



Cut into individual pieces and see what shapes you can make

Can you make the square again? How about any animals? Or letters in the alphabet?

ami

Countdown

This game can be played by any number of people

Pick any 5 small cards and 1 large

2	7	3	7	4	8	10	5	9	4	25	100
8	1	10	3	6	9	6	1	2	5	75	50

These are the game cards.

2	10	4	6	9	50	**Game cards**

Pick any other 3 small numbers. These combine to make the target (341)

3	4	1	**Target**

Everybody now has 30 seconds to see who can get closes to the target number using the game cards at most once each, and operations **+ - x ÷**

$$50 \times 2 \times 3 = 300$$

$$10 \times 4 = 40 \quad +$$

<u>340</u> Close but not exact!

Pell Numbers

It can be proven that there are no integer (whole number) solutions to

$$\sqrt{2} = \frac{a}{b}$$

So there are no integer solutions to

$$2b^2 - a^2 = 0$$

But can you find positive integer values of a and b that *nearly* work? That give the answer 1 or -1?

$$2b^2 - a^2 = \pm 1$$

Can you find a pattern to all your solutions?

What does $\frac{a}{b}$ give a good approximation to?

Balls and Books

1. You have a bag of 5 balls, all of different colors. You take two balls out of the bag. How many different possible outcomes are there?

What if you add another ball of a new color to the bag (so you have 6)? What about 7 balls?

Is there a similar problem you've already solved?

2. You have 2 blue books and 3 green books. The blue books all look the same and the green books all look the same.

How many ways are there to line them up in the shelf?

What if there are 4 green books? 5 green books?

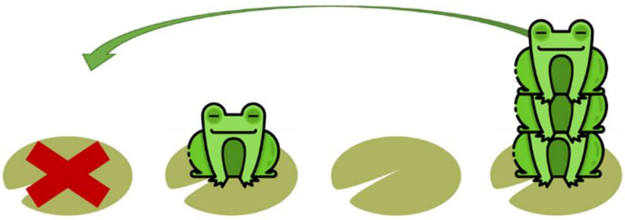
Frog Party

5 frogs want to have a party. To do this they must all be on the same lily pad.

Frogs can jump left and right however there are a couple of rules:

1. Frogs jump together, so that 1 frog can jump 1 space, 2 frogs can jump 2 spaces etc.

2. Frogs cannot land on an empty lily pad



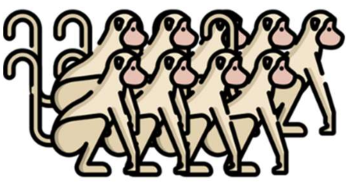
Can you find a way to move the frogs so they can have a party together on the same lily pad?

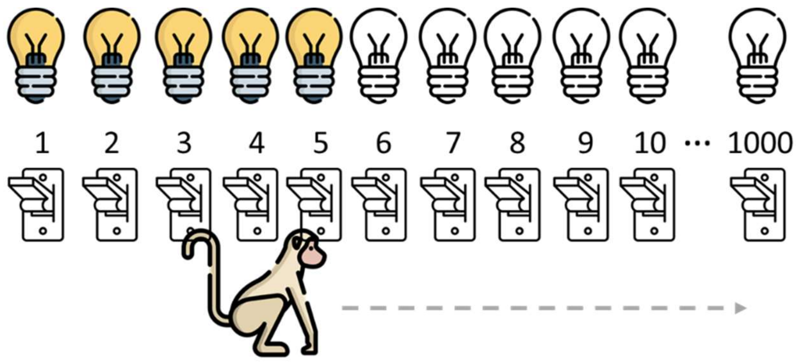
Is it possible to have a party on every lily pad?

Monkey Business

A large room has 1000 lightbulbs in it, all are switched off, but each has its own switch to turn on

1000 monkeys enter the room and decide to press the light switches in a very particular way.





The 1st monkey presses every multiple of 1.
The 2nd monkey presses every multiple of 2.
The 3rd monkey presses every multiple of 3.
Etc., until the 1000th monkey.

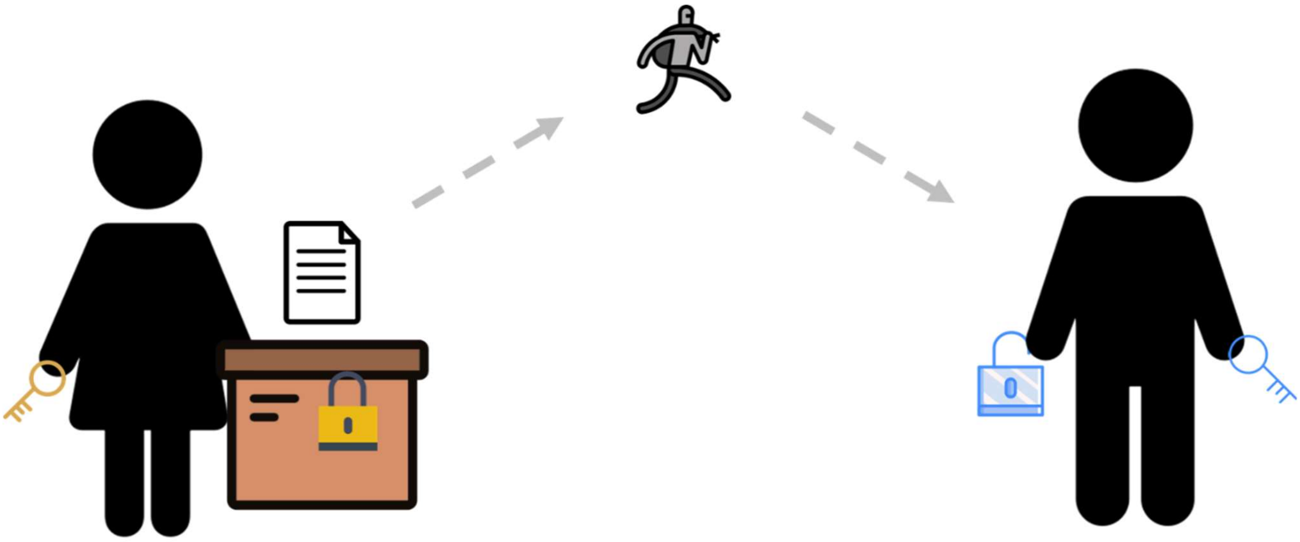
After all the monkeys have finished pressing switches:

1. Will light number 10 be on or off?

2. How many lights in total will be on?

Locks and Keys

Alice has a secret message that she wants to send to Bob, who lives far away. She has a box and a lock and one key. Bob also has a lock with one key.

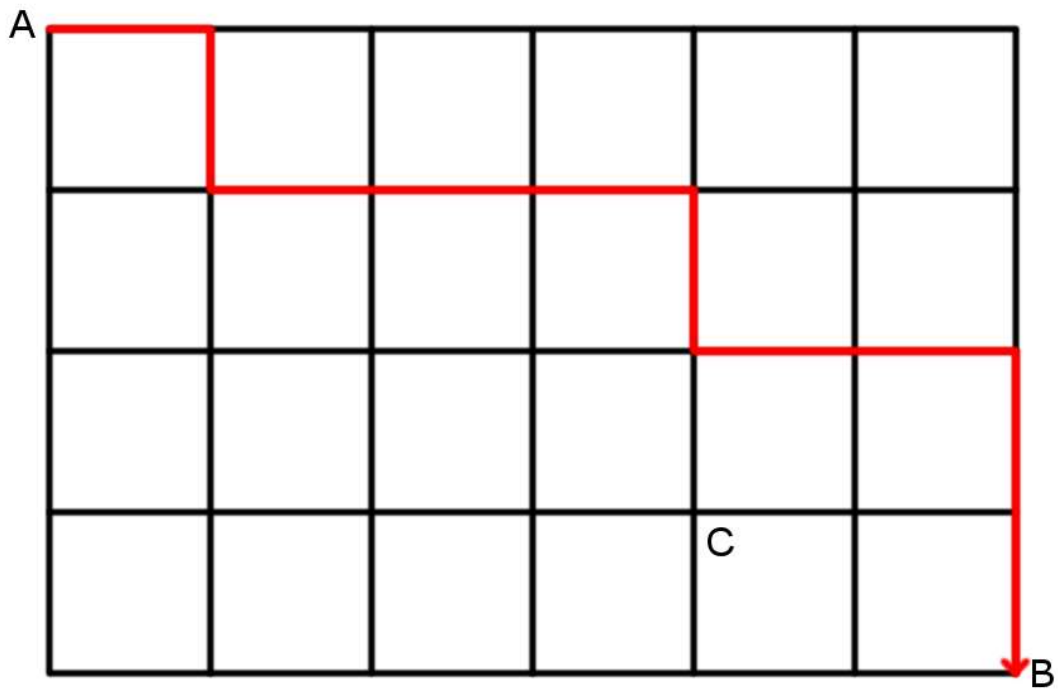


To send things between each other Alice and Bob have a messenger, who can travel multiple times between them. But they don't trust him, and don't want him to read the message.

How can Alice send her message to Bob without the messenger reading it?

Paths

1. How many ways are there from A to B in the grid if you can only walk along the lines and only walk right or down each time? How about from A to C? How about from A to any other point?



2. How can you write down a path from A to B as a sequence of 10 letters? Or as a set of 4 numbers?

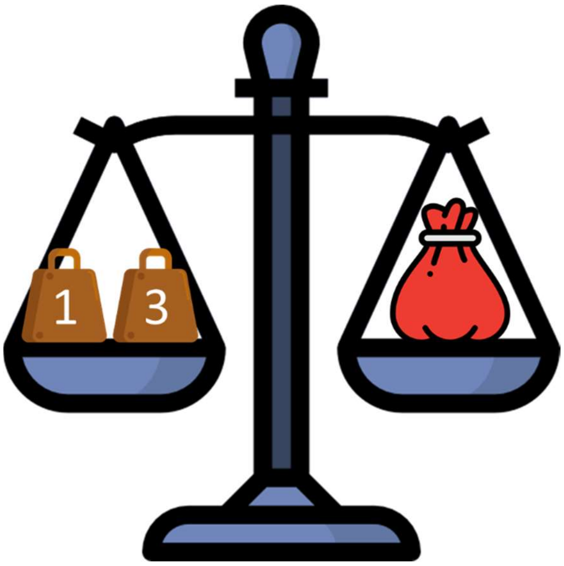
Only try these after solving 1 and 2

3. How many ways are there to line up 4 blue and 6 green books in your shelf?

4. If you take 3 balls from a bag of 7 different balls, how many possible different outcomes are there?

Scales and Weights

You have a shop selling maize flour by weight. Your customers may want to buy any amount between 1 and 40 kg of flour.

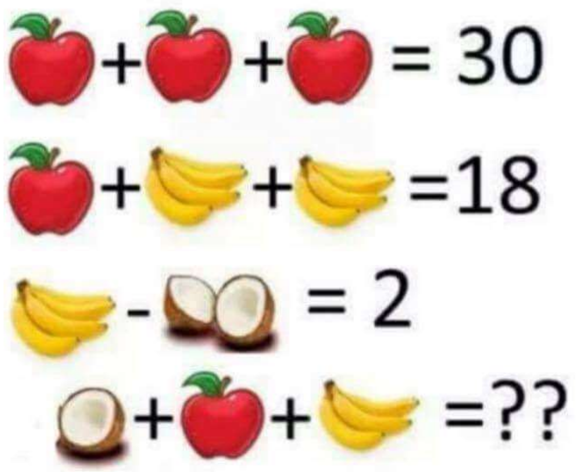


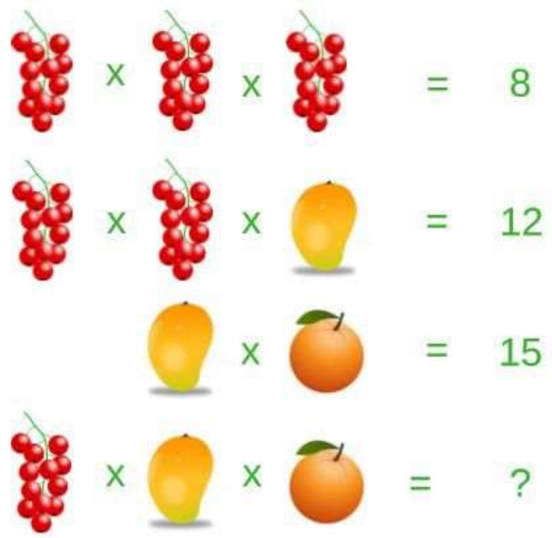
You also have a balancing scale and want to buy weighing stones to use with it.

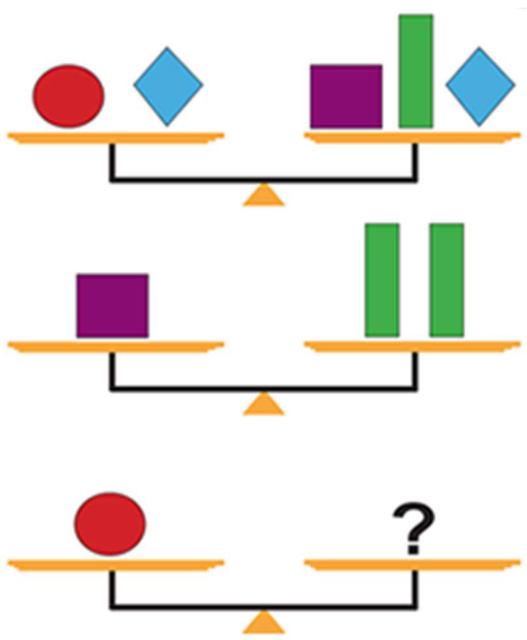
Which weighing stones should you buy so you can measure any weight between 1 and 40 kg? (You want to buy as few as possible)

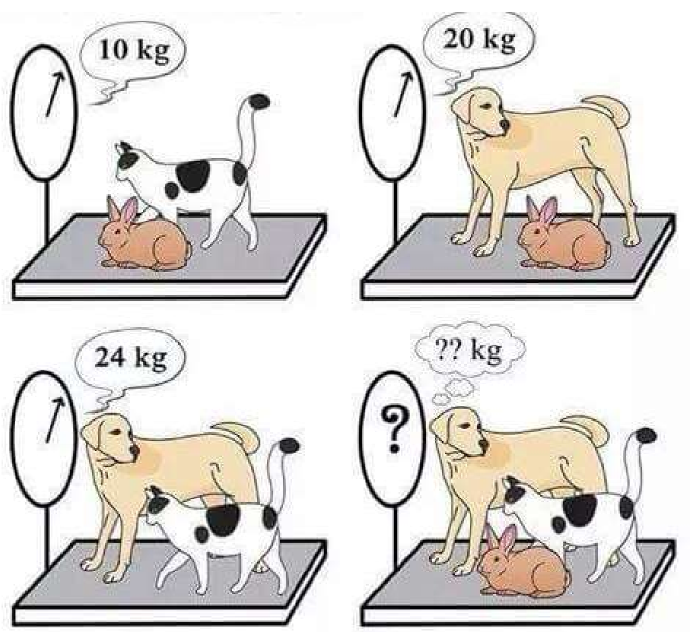
Picture Puzzles I

The following 4 problems all have pictures to represent numbers. Can you figure them out?









Make Many

Task 1

How many different three digit numbers can you make using only the numbers 100, 2 and 3 at most once and as many of the operators $+ \; - \times \div$ and brackets as you like?

e.g. $100 \times (2+3) = 500$

$100 \times 3 = 300$

$100 + 3 + 2 = 105$

Task 2

Using the numbers in the blue box, try and make all of the three digit target numbers. Hopefully Task 1 will have made you think of some tricks for the harder ones!

5 5 6 7 50 100

635

667

665

785

564

420

202

419

Secret Santa

A group of friends want to buy one present each and everyone wants to receive one present (Secret Santa). The rules of a Secret Santa are that each person's name is put in a hat and the names are mixed. Then each person must choose 1 name from the hat. If you choose your own name, you must put it back in the hat (and all start again if necessary!).

If **2 people** do a secret santa there is only one way: Person A gives to person B and person B gives to person A.

3 people: With 3 people, there are 2 possible ways. Can you think why?

4 people: Now how many different ways are there with 4 people?

The final challenge is to find the number of different scenarios with **5 people**.

Tic Tac Toe with levels

Introduction

This game is best played after playing the simpler Tic-Tac-Toe (G1 sheet). This is a variation of Tic-Tac-Toe that has much more variation. This game is designed to get students to think about having a strategy to win the game. After students have played the game for a while ask them some of these questions.

- What is the best way to play this game?
- Is it better to go first or second?
- Where is the best position to start from?
- Are the rules the only thing you need to know to win the game or is there something more?
- What is strategy and how does it relate to this game?
- What are some of the strategies to playing this game?
- Should you play this game randomly or should you think about your moves?

If you have previously played original Tic-Tac-Toe, ask the students what are the similarities and differences between this game and Tic-Tac-Toe. Some questions to discuss include:

- Which game is easier to win? Why?
- Which game requires more thinking, and why?
- Are the strategies you thought of for both games similar in any way? How?

Student should realise that this game is much more complex than original Tic-Tac-Toe which has a clear strategy to avoid losing. In fact, even a computer analysing the game could not find a winning strategy for Tic-Tac-Toe with levels. This should show students how even simple games can require lots of mathematical thinking, strategy and logic.

Extension

Add another row so you now have a 4 x 3 grid. Use the same rules. Does this make the game harder or easier?

Next thing to do

After doing this activity or in your next session you can try the activity 15 game, original Tic-Tac-Toe or Letter matching, which are all games related to this game.

Letter Matching

Introduction

You should only play this game after you have played Tic-tac-toe!

This games requires that you to keep track of yours and your opponents words, think ahead and predict what your opponent will do. Like Tic-tac-toe you need to develop a strategy if you want to win this game.

It is usually harder for students to develop a strategy for this game than for Tic-tac-toe. Ask the students whether they think this game is easier or harder than Tic-tac-toe, and why.

Solution

Discuss with the students the differences between playing the game in a line and playing in the square. Discuss these questions.

- Is it easier to play the game in the square or in a line? Why?
- Do you get more wins, losses or draws?
- What is special about the arrangement of the square?
- Is playing in the square similar to anything you have played before?

If you have also played 15 game you can see if students notice the similarities with this game (they are basically the same game, called isomorphic games). After playing in the square a few times, students should realise that to get three words that share a letter they just need to get three words in a line. This is because if you look at the words in each horizontal, vertical and diagonal line, they all contain a common letter.

	eat	bee	less	→e
	air	bits	lip	→i
	soda	book	lot	→o
↙	↓	↓	↓	↘
s	a	b	l	t

That is why this is a special arrangement of the letters!

We have now transformed the game into getting a line of 3 words. This means that it is basically the same game (isomorphic) as **Tic-tac-toe**!

Students usually say that Tic-tac-toe is an easier game to play than this game, but we have shown that we can transform this game so that we are basically playing Tic-tac-toe again and made the game easy.

In mathematics we often change the way we look at things to make them easier for us. It is a very powerful tool. Here we have transformed a hard game (Letter matching) into an easy one (Tic-tac-toe). With this knowledge, students should be much better at Letter Matching now.

Extension

Can students come up with their own words to play this game? They must be able to write them in a square like above for the game to work!

Counting Chickens

Introduction

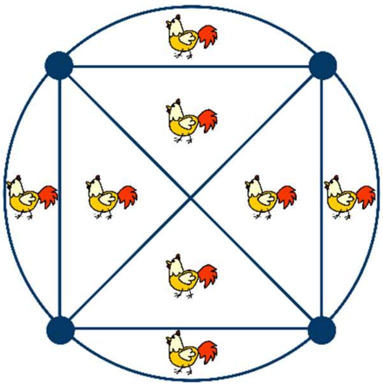
This activity looks at the interesting concept of having a circle with n points on its perimeter and what happens when you joint those points up.

Three examples are given on the question sheet so students can see how to visualize the problems. The activity questions rely on students being able to develop on these primary examples to solve more complicated cases. The extension looks at spotting a pattern and developing a rule.

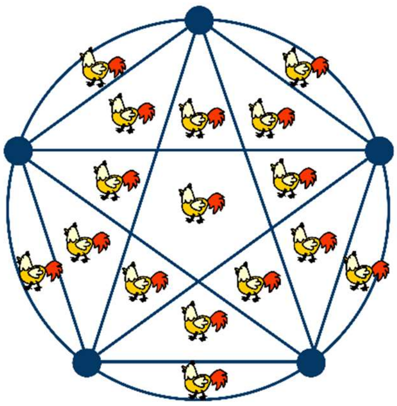
It is advisable that students try drawing out each case. It should be noted that the use of colours can be a way for students to picture the different regions more easily.

Solution

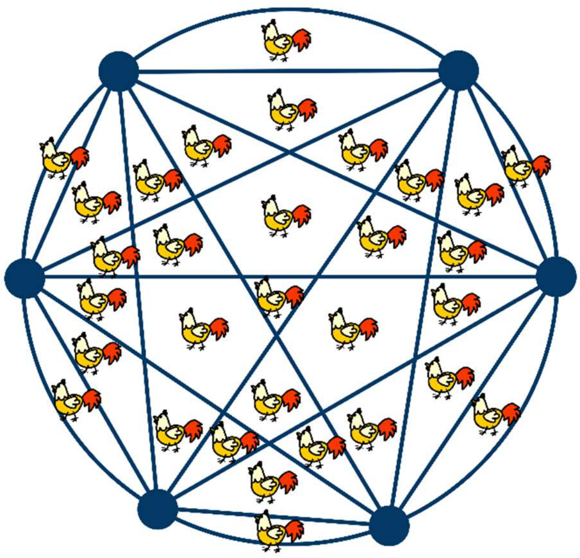
1) With 4 fence posts the farmer can keep 8 chickens.



2) With 5 fence posts the farmer can keep 16 chickens.



3) With 6 fence posts the farmer can keep 31 chickens.



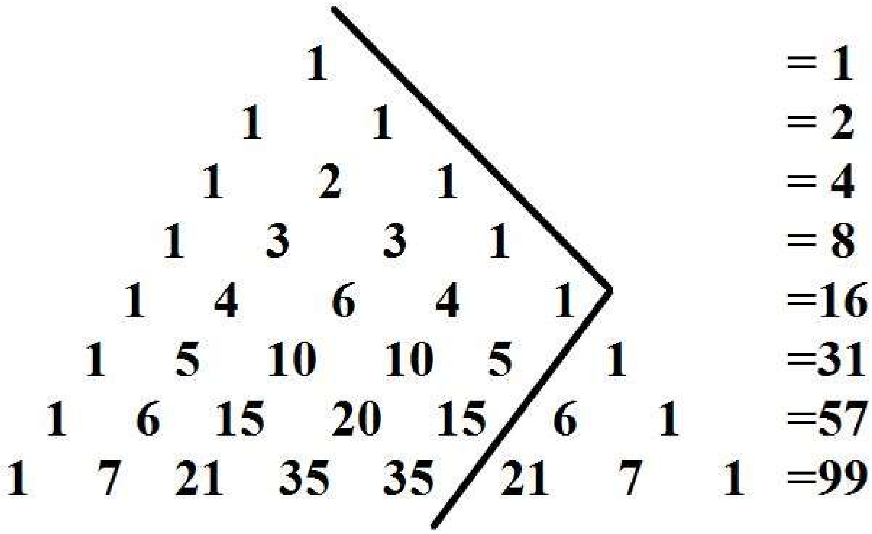
Extension

Is there a pattern between the number of fence posts and the maximum number of chickens that can be kept?

Hint: Fill out this table (some of it has been filled out for you)

Number of fence posts	Number of chickens
1	1
2	2
3	4
4	?
5	?
6	?
7	?
8	?

Using Pascal's triangle, what will happen if you add the numbers to the left of the line in each row? Can you spot the pattern now? Can you explain this strange pattern?



Pell Numbers

Introduction

This activity starts with trial and error to find some values of **a** and **b** that work. Set out any correct answers that students get in a table like this, putting the smallest solutions at the top.

b	a	Answer

Students will hopefully spot some patterns appearing. If you have access to a computer or tablet, you could try to generate more solutions on a spreadsheet such as GeoGebra or Excel.

Solution

b	a	Answer
0	1	-1
1	1	1
2	3	-1
5	7	1
12	17	-1
29	41	1
…	…	

You can find the next row for **b** by adding the previous answers for **a** and **b** together.

You can find the next for **a** by adding the previous answer for **a** and the new value for **b**.

e.g. **29 = 12+17** and **41=29+17**

You can also find new values for **a** and **b** by adding together twice the previous term and the term before that.

e.g. **29=2x12+5** and **41 = 2x17+7**

Mathematically, for the **b** column, this is written as $\boldsymbol{b_n = 2b_{n-1}+b_{n-2}}$ where $\boldsymbol{b_0 = 0}$ **and** $\boldsymbol{b_{1=}1.}$

$\boldsymbol{\frac{a}{b}}$ for each row gives an approximation to $\boldsymbol{\sqrt{2}}$. The approximation gets better as you go down the table. But it will never be perfect no matter how long you do this for!

Extension

Try doing the activity again but starting with $\boldsymbol{\sqrt{3} = \frac{a}{b}}$. More info can be found at https://en.wikipedia.org/wiki/Pell_number. Thanks to Kevin Buzzard from Imperial College for introducing SAMI to the activity.

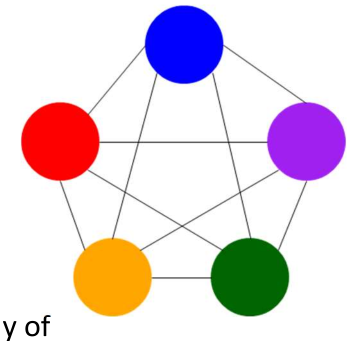
Balls and Books

Introduction

This is a follow-up to “Handshakes”. The idea is to show different ways of thinking about a problem, and realize how seemingly different problems can be related to each other. You should encourage students to be systematic and decide on a precise and ordered way to describe what has been selected.

Solution

1. There are different ways to approach the first problem. One is trying to list all the possible combinations. A good question to ask is: **how to write down a single combination?** One could write down two letters, the letters being the initials of the colors (e.g. BY for “blue and yellow”). Or assign numbers to the colors and write down these.



Another way is to draw each of the balls on paper, and connect possible pairs (this way it is clear that the problem is the same as the handshake problem).

Either way, one obtains **<u>10 different combinations</u>** for 5 balls.
(12, 13, 14, 15, 23, 24, 25, 34, 35, 45).

To solve the problem for more balls, one can ask the following question: **Which new combinations arise when adding another ball?** The new ball can be combined with any of the 5 previous balls, giving 10+5=15 solutions for 6 balls. Similarly, we get 15+6=21 solutions for 7 balls.

A different way of solving this problem is realizing that each of the 5 balls can be combined with each of the other 4 balls, giving 5*4=20 combinations. However, doing this we get each combination twice (e.g. we get 12 and 21, which are the same). So we need to divide the answer by 2 to get 20/2=10.

2. Again, ask the two questions:

How to write down a single combination? The most obvious way is to write down a string of letters representing the color, e.g. (BGGBG).

A different way is to write down the two positions where the 2 blue books are. In the example, they would be in positions 1 and 4, so write 14. This is less obvious and it's fine if the students don't find it. If they do however, they will see that this problem is the same as the first one, and so there are **10 possibilities**!

Which new combinations arise when adding a 4th green book? There are two kinds of line-ups to consider: Those where the last book is green, and where it is red. If the last book is green, then removing it gives us a line-up for 2 red and 3 green books. We already know there are 10 of them! If the last book is red, then there are 5 different possible positions for the other red book. Thus there are 10+5 combinations for 2 red and 4 green books. Similarly, 21 for 2 red and 5 green books.

Extension

Consider the first problem again, but this time taking 3 balls out each time.
Try to decide a general rule or formula for any of the problems (e.g. picking 2 balls from N in a bag)

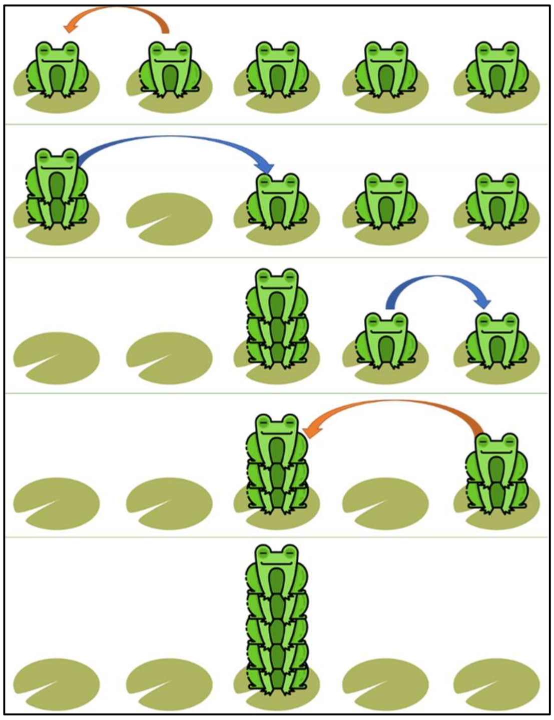
Frog Party

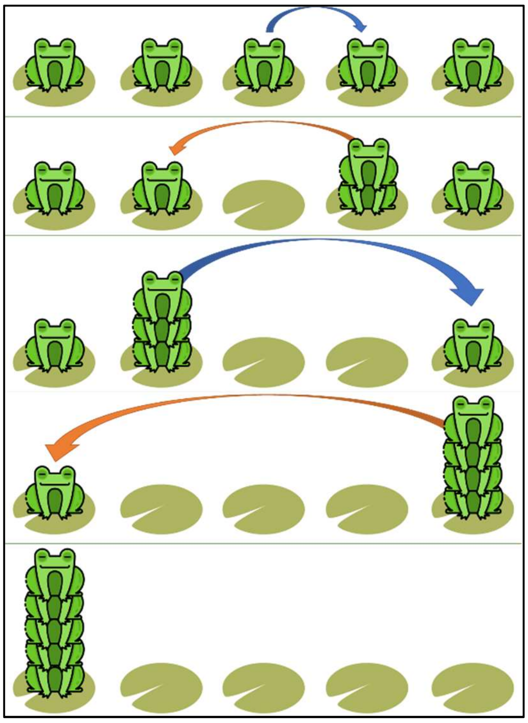
Introduction

This is a really fun problem, and can be used to build many much harder problems. It is important when starting to problem to make sure the rules are clear. You might want to do this by showing examples of moves which are and are not allowed, using coins on top of pieces of paper or similar.

Solution

There are many ways to solve this, and it is possible to have a party on every single lily pad. Two examples are given below, however you should ask students to also share their solutions





Extension

There are lots of ways you can make this problem more challenging and interesting, which include:

- Add more lily pads, is it still possible with 6, 10 or N?
- Choose one of the frogs to be the Queen frog, who must be at the top of the party
- Swap one of the frogs for a lazy toad, who refuses to move. Is it still possible?

Note, the strategy used in the second solution of working from the inside out also works for any number of lily pads!

Monkey Business

Introduction

This is another problem where the solution seems impossible because there are too many things to consider (1000 monkeys!), but again becomes achievable once simplified and able to see patterns. To start the problem you should ensure students understand the rules, perhaps imitating with 4 students and the same number of coins or cards that can be flipped to face up or down.

The first student flip all cards up (u):	[u, u, u, u]
The second student flip the evens down (d):	[u, d, u, d]
The third student would flip card 3:	[u, d, d, d]
The fourth student would flip card 4:	[u, d, d, u]

Solution

1. For the first part of the problem you only need to consider 10 monkeys (as the 11^{th} monkey and later will not press any of the first 10 switches). So which monkeys will press switch number 10? Well, as 10 is divisible by 1, 2, 5 and 10 there are exactly 4 monkeys which will press the switch

- Monkey 1 turns it on
- Monkey 2 turns it off
- Monkey 5 turns it back on
- Monkey 10 turns it back off

So switch 10 will be **off**.

2. The next part is more challenging, but again relies of thinking about how many monkeys will press each switch.

Switch 10 was off because every time one monkey turned it on another turned it off. We were able to split the number 10 into factor pairs (1x10, 2x5), which will always result in the light being turned off.

Most numbers will be pressed by an even number of monkeys, e.g. 24 has factors (1x24, 2x12, 3x8, 4x6), so monkeys 1, 2, 3, 4, 6, 8, 12, 24 will press the switch, and as this is an even number the light will be off.

Square numbers are the only ones which do not, e.g. 16 has factors (1x16, 2x8, **4x4**). Monkeys 1, 2, **4**, 8, 16 will press the switch, and as this is an odd number the light will stay on (monkey 4 won't press it twice!).

So we just need to work out how many square numbers there are between 1 and 1000?

If we think about the larges square number less than 1000, **30x30=900, 31x31=961**, **32x32=1024**

So there will be **31 switches left on**! (switch numbers 1, 4, 9, 16, 25, 36, … , 900, 961)

Extension

How many lights would stay on if only the even numbered monkeys decided to take their turn pressing the switches?

Locks and Keys

Introduction

This problem is not specified very precisely, and in this case it is intentional. The aim of this problem is to work with assumptions. The problem has different solutions under different assumptions, and it might be easy or even impossible depending on the assumptions. For example, if we assume that the messenger can break the box or the lock, then the problem is impossible (and somewhat pointless).

There is no right or wrong solution because there is no right or wrong set of assumptions. However, if we specify our assumptions clearly, then for the given set of assumptions any solution is either right or wrong. The facilitator should encourage students to decide on what should be allowed or not so that the problem is not solved too easily or made impossible.

This activity involves lots of discussion between groups. It could help to have paper cutouts of locks, keys and boxes so students can demonstrate rather than only explain. You should encourage students to demonstrate their solutions, and to challenge each other to see if they can find a way to intercept.

Solution

There's are lots of different strategies that could be tested and argued, here we will only outline a few possibilities, it is up to the students and facilitator to decide if their own solutions work or not.

We will start with a few assumptions:

1. The messenger cannot break the box or the lock.
2. The messenger cannot make copies of keys.

Case 1: Alice locks the box and sends it to Bob. Bob sends the messenger back to Alice. Alice now gives the messenger the key to send it to Bob. The messenger gives the key to Bob and Bob unlocks the box.

Intercept: The messenger only pretends to deliver the box to Bob, but instead keeps it. He then returns it back to Alice and obtains the key to unlock the box and read the message.

Case 2: Alice sends the key using a different messenger than for the box.

Intercept: The messengers collaborate. You would need to decide in the rules whether this would be possible.

Case 3: Alice locks the box with her lock and sends it to Bob. Bob attaches his own lock and sends it back. Alice takes her lock off the box and sends it back again. Finally, Bob takes his own lock off to read the message.

Intercept: In this case there would be no way to intercept the message without breaking the box or copying keys!

Extension

Bob opens the box and see it is instructions to play a game. To start the game Alice and Bob will flip a coin to decide who goes first, however they will still need the messenger to communicate the outcome. Who should flip the coin? Who should choose heads or tails? How can they communicate in a way so that neither person can cheat?

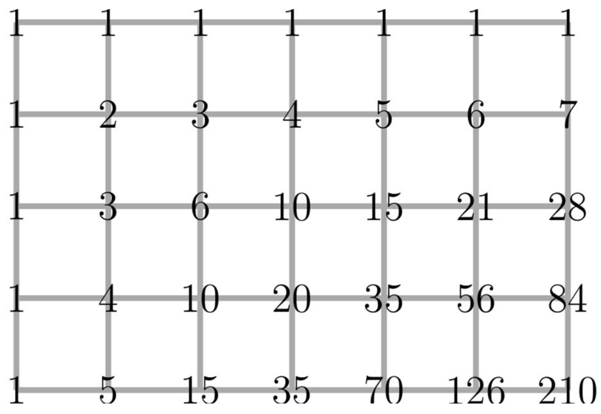
Paths

Introduction. This is a follow-up to “Balls and Books”. It is quite challenging as it introduces Pascal's triangle and using Pascal's triangle to solve other combinatorial problems. Students will very likely need some guidance (in the form of nudges and simpler intermediate tasks) to solve the first problem.

Solution

1. Problem 1 might look scary at first. Trying to list all possible path will be hopeless, as we'll see there are too many. **Encourage the students to be systematic and solve smaller problems to points that are closer to A first!** A good task to **write, for each point on the grid, the number of paths to that point.** For some points this will be easy, for example for the points in the first row or first column there is only 1 path. For the points in the second row or column this is a little harder, but still possible. For a point in the second row, you can only go down once. If the point is in the third column for example, then there are 3 choices where to go down: Either in the first, second or third column. Thus, in the second row the numbers will be: 1, 2, 3, 4, 5, 6, 7.

From here on, it gets much harder. One question to lead onto the right track is: **Can you get the number of paths from A to B, if you already know the number of paths from A to all the points closer to B?** The key realization is that if you want to know the number of paths to a point X, and you know the number of paths for the point above X and for the point to the left of X, then the number of paths to X is simply the sum of number of paths for the point above X and the number of paths for the point to the left of X! This way, it is easy to fill in the grid by simply adding up numbers. It will look like this. **From A to C there are 42 paths, from A to B there are 210 paths**.



2. Some students might have already solved problem 2 when trying the first problem. Considering one can only go **D**own and **R**ight, the natural thing to do is simply writing down a sequence of R's and D's. The example path would be RDRRRDRRDD. Writing it down as numbers is slightly less straight-forward. One could write down the columns where the path goes down, however this will not help with solving the next problem. Instead, we can **write down only which path segments are going down**. In our example, the 2nd, 6th, 9th and 10th segment goes down, so we'd write 2, 6, 9, 10.

Extension

A) If students have already done the “Balls and Books” problem you could ask to try and use the same methods to solve the problem of arranging 4 blue and 6 green books on a shelf
Note: Instead of using RDRRRD, you are now using book colours BGBBBG – see the similarity! The answer will be 210 again

B) Attempt the extension activity from “Balls and Books”, to calculate the number of ways of taking 3 balls out of a bag with 5 different colours.
Note: If you numbered the balls you are considering how many ways there are of picking 3 numbers from 5 (e.g. 123, 145, 234 etc.). This looks very similar to the situation above if had 5 columns (representing the colours), and could take 3 down steps (picking a colour). The solution in this case will be 35!

Scales and Weights

Introduction

The statement is not 100% precise, so: You want to put all the flour in one bag and weigh it, not splitting it into smaller bags for weighing. The weights to measure are whole numbers only.

Solution

This problem has two different versions depending on the assumptions you're using. Both are interesting, and it is worth **encouraging students to try both.**

- The first version is probably slightly easier, and assumes that you can only put weighing stones on the side of the scale.
- In the second version, you can put weighing stones on either side, giving more options.

Encourage students to try for much smaller numbers than 40 first. For example, if we only want to measure weights up to 7kg, then weighing stones of 1 kg and 2 kg and 4kg are sufficient (check that you can measure 1,2,3,4,5,6,7kg with these!).

If we could put weights on both sides we could come up with more interesting solutions, for example if we had 1kg, 3kg and 6kg weights we could weight 1,2,3,4,5,6,7,8,9, and 10!

E.g. If we have 8kg of flour, we could add the 1kg weight to it and it would balance against the 6kg and 3kg. See if you can make all the other numbers!

The best strategy for the first problem is to start from 1kg, and every time there is a number you can't make add another stone. Quickly you see that if you have **1kg, 2kg, 4kg, 8kg, 16kg**, you can measure every number up to 31kg. You will therefore need a 5^{th} stone to reach 40kg. You could make this a 9kg stone, however it would be better if you added a **32kg** stone and now can measure everything up to 63kg.

For the second problem you not only want to consider the total when stones are added together, but the difference when they are subtracted (because you can have stones on either side). You want to give as many different numbers as possible using as few stones as possible.

To start we will either need a 1kg stone, or 2 stones with a 1kg difference. In the example above we started with a 1kg stone but what if we started with **1kg** and **3kg**? We can now measure 1,2,3,4.
To get 5kg we can pick the largest number which will balance against the 5kg flour + 1kg + 3kg stones, so we should pick **9kg**. This now lets us make every number up to 9+3+1 = 13kg. To get 14kg we simply add a **27kg** stone, because 27kg – 9kg – 3kg – 1kg = 14. With the 27kg stone we can now measure everything up to 40Kg, so **it is possible with only these 4 stones!**

Extension

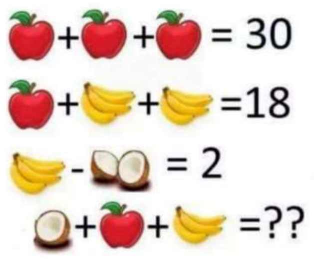
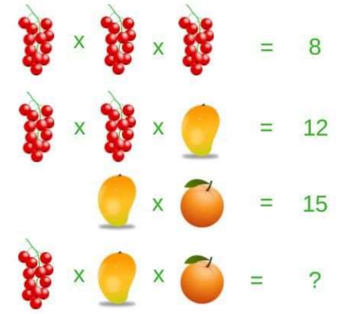
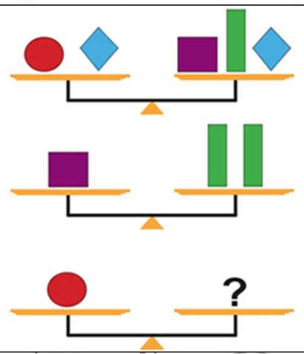
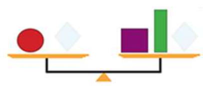
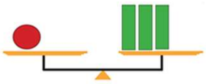
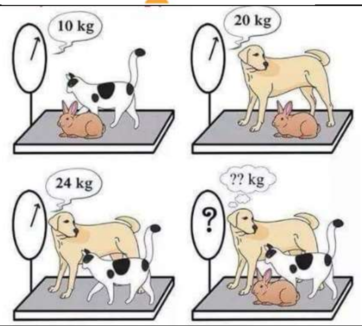
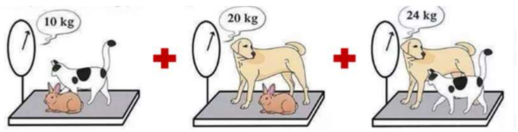
Continue this process to see which stones you would use up to 1000kg. Is there a formula you could use to calculate?

Picture Puzzles I

Introduction

These problems can all be solved in different ways, but it is recommended you encourage students to use logic and reasoning skills instead of guessing numbers. Usually each line gives you a piece of information that you can use in the next, so you should think carefully about each piece of information before moving onto the next. The problems also get harder as you go, so encourage students to start with the top-left puzzle first.

Solution

	3 apples = 30, so **1 apple = 10** 10 + 2 bananas = 18, so **1 banana = 4** 1 banana – 1 coconut = 2, so **1 coconut = 2** Altogether you get 2 + 10 + 4 = **<u>16</u>**
	<u>2</u> x <u>2</u> x <u>2</u> = 8 2 x 2 x <u>3</u> = 12 3 x <u>5</u> = 15 2 x 3 x 5 = **<u>30</u>**
	Remove the diamond from both sides and the balance will still be level, so the circle is the same as one square and one rectangle  The second line tells us that the square is the same as 2 rectangles so we could replace in the square in the image above 
	Add all the animals in the 3 known scales together  This will give you **2 cats, 2 dogs, and 2 rabbits** with a total weight = **54kg** Therefore **1 cat, 1 dog and 1 rabbit** will have a total weight **27kg** (you don't need to work out all the individual weights!)

Extension

Ask students to try and make their own picture puzzles!

Make Many

Introduction

This type of puzzle comes from a TV show. The first task should help students practice techniques to use in Task 2. Students might be surprised at how many numbers you can make using only 100, 2 and 3. The full version of the game is presented in the game 'Countdown' (page 19)

Students could start on their own and then share their answers to see if as I group they can find them all. Don't tell them there are 19 until they get there!

Solution

Task 1

100, 101, 102, 103, 105, 106, 150, 194, 197, 200, 203, 206, 294, 298, 300, 302, 306, 500, 600
Some of the tricky ones are, e.g. 194 = $(100 - 3) \times 2$

It shows how useful brackets are!

Task 2

Here is one way to solve each problem. Students may come up with other correct solutions – just check they have only used each number once!

$635 = 6 \times 100 + 7 \times 5$

$667 = 6 \times 100 + 50 + 5 + 5 + 7$

$665 = 7 \times (100 - 5)$

564 =$6 \times (100 - 7 + \frac{5}{5})$

785 = $7 \times 100 + 50 + 5 \times 6 + 5$

202 = $= (7 - 5) \times (100 + \frac{5}{5})$

420$= 6 \times 7 \times 100 \div (5 + 5)$

$419 = (7 \times 6 \times 50 - 5) \div 5$

Extension

Have a competition following the game rules on page 19.

Secret Santa

Introduction

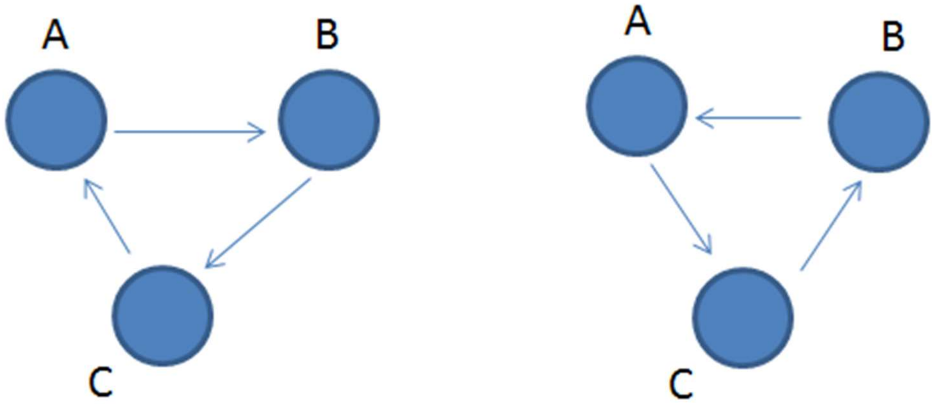
In this problem, we are trying to count the number of possible ways to have a secret Santa. The idea behind secret Santa is that the names of all the people involved are put in a hat and each person has to pick one name from the hat, not including your name.

Each stage gets progressively harder in the hope that the students start to understand the general rules behind the way to find the number of possible scenarios.

Firstly, let the students think about ways to represent this problem. The students could even arrange themselves in groups to reenact different scenarios and count the number of different ways of having a secret Santa.

Solution

<u>3 people</u>

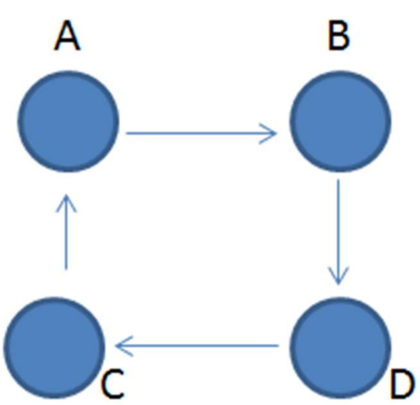


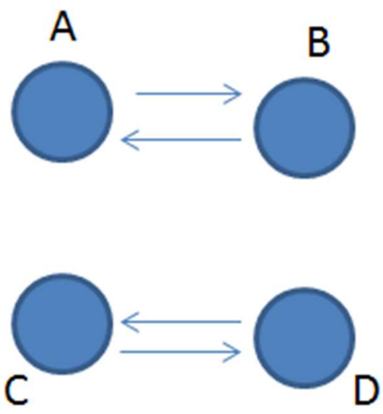
In this problem, you might have noticed that if A could either give to B or C, thus there are the 2 circular solutions above.

3 people, 2 ways

<u>4 people</u>

For 4 people, if you looked at the possibilities using networks, you might have realised that the network can be decomposed into 2 networks with 2 people in them:

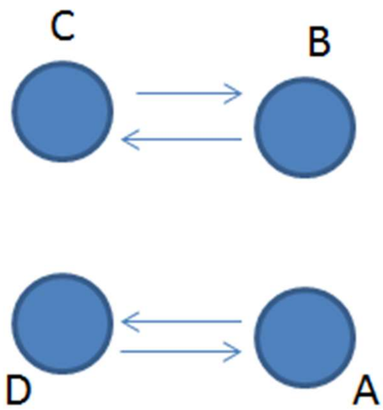


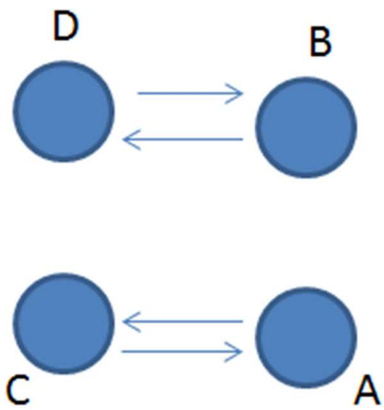


The problem is therefore how many different ways do you have of having everyone in a same network and having 2 smaller networks with 2 people each?

The answer is that for the larger network, there are 3! (= $3 \times 2 \times 1$) ways of doing it as there are 3! different ways of changing the order of neighbouring nodes. You could write all these options as a list that you can imagine into a circle: ABCD, ABDC, ACBD,ACDB, ADBC,ADCB.

For the second question, we can see that there are 3 possibilities, which are the one above and the two below:





Therefore, for 4 people, you have 6+3 = 9

4 people, 9 ways

<u>5 people</u>

There are two different options. A circle of 5, which could be done in 4! = $4 \times 3 \times 2 \times 1 = 24$ ways.

A circle of 3 and a pair. The 3 could be chosen in 10 different ways and then there are two ways round the circle so that is 20.

5 people, 44 ways

Extension

How about 6 people? 7? There are some properties of the sequence 2, 9, 44, etc. here https://oeis.org/A000166