

Frogs – Background Notes

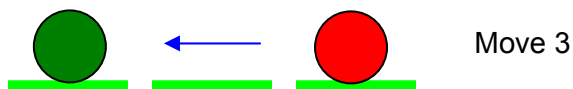
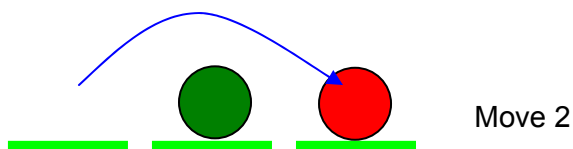
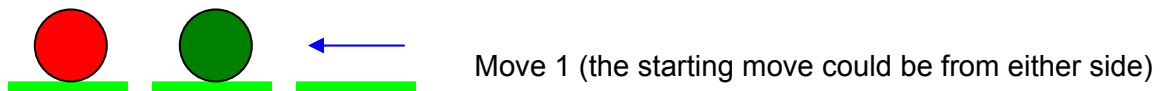
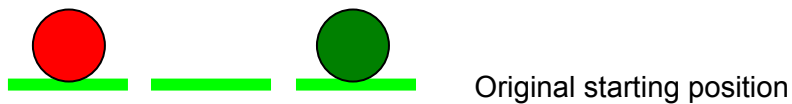
The results may be given in several ways, often depending on the ability of the child. It is important that the children are encouraged to use their own process and strategies to investigate the situation, but the teacher must bear in mind that this may lead to alternate ways of expressing the solution.

Process/strategy

- Be systematic

Start with the minimum number of frogs and build up. The easiest way to start would be have the same number of frogs on both sides.

1 red 1 green

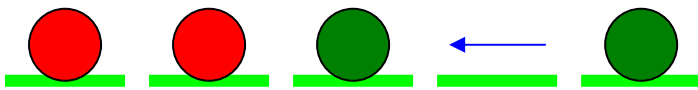


Minimum number of moves for 1 red frog and 1 green = 3
(2 slides and 1 jump)

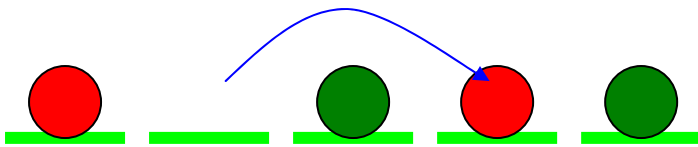
2 red 2 green



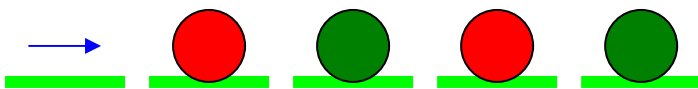
Original starting position



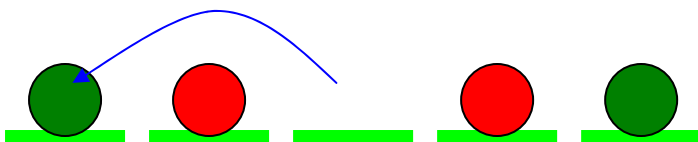
Move 1



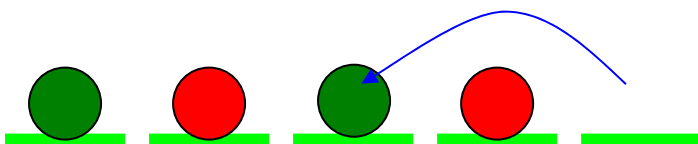
Move 2



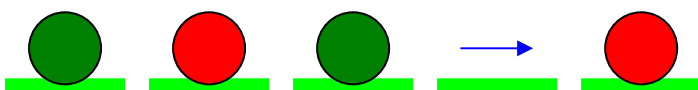
Move 3



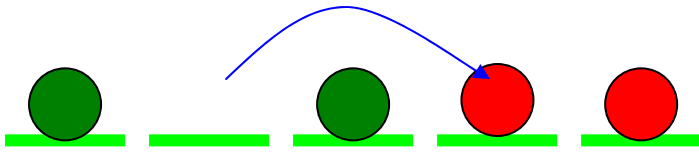
Move 4



Move 5



Move 6



Move 7



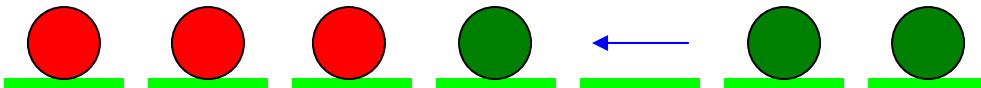
Move 8

Minimum number of moves for 2 red frogs and 2 green frogs = 8
(4 slides and 4 jumps)

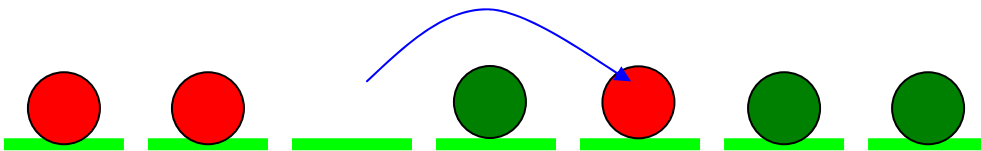
3 red 3 green



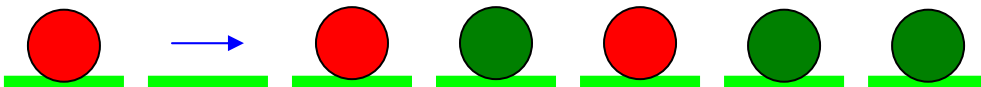
Original starting Position



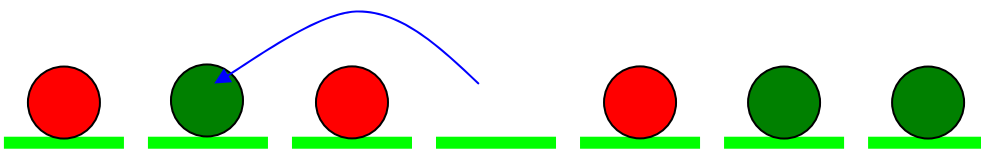
Move 1



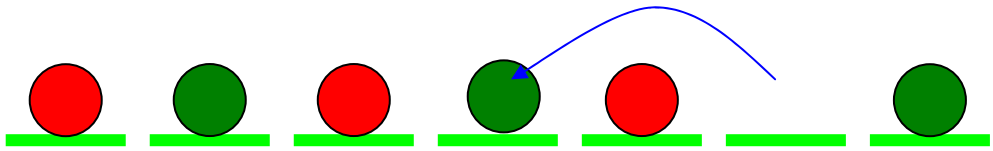
Move 2



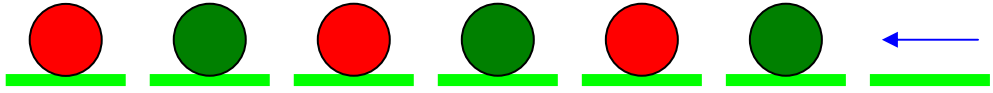
Move 3



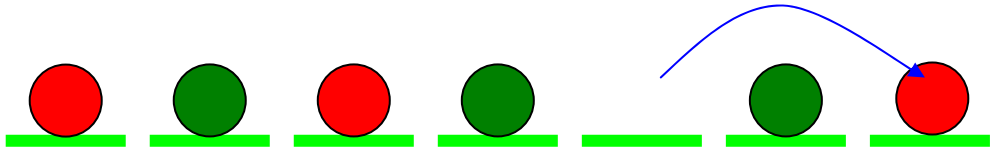
Move 4



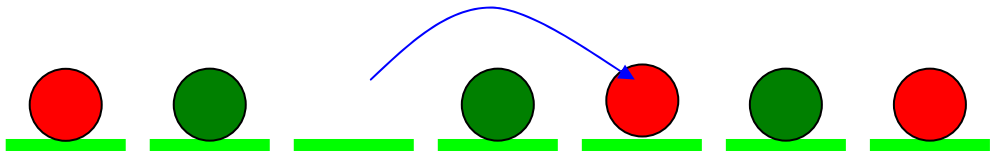
Move 5



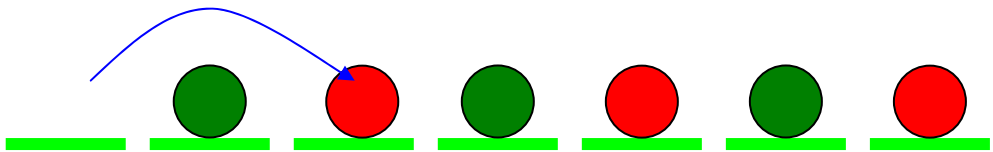
Move 6



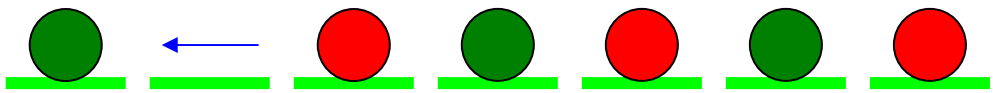
Move 7



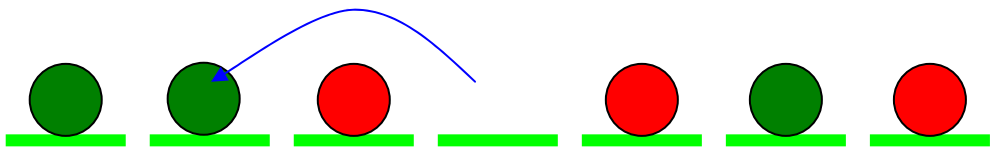
Move 8



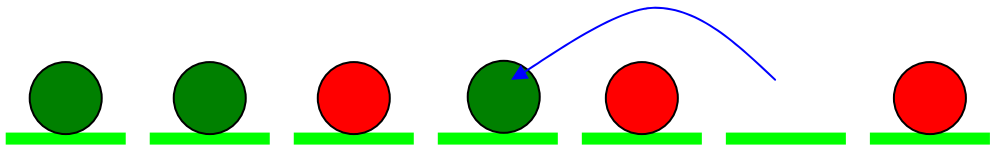
Move 9



Move 10



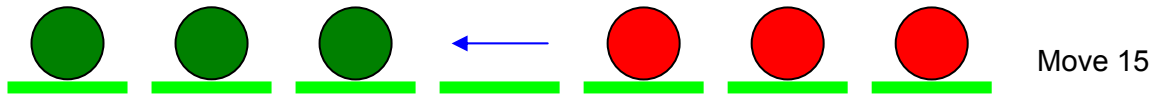
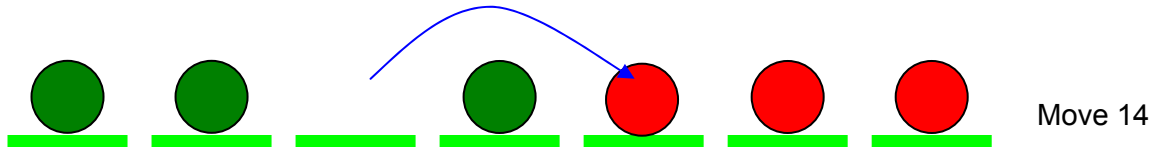
Move 11



Move 12



Move 13



Minimum number of moves for 3 red frog and 3 green = 15
(6 slides and 9 jumps)

- Tabulate results

First we will tabulate the results for the same number of frogs on each side.

Number of frogs		Minimum number of moves		
Red Frogs	Green Frogs	Slides	Jumps	Moves
2	2	4	4	8
3	3	6	9	15
4	4	8	16	24
5	5	10	25	35
6	6	12	36	38
7	7	14	49	63
8	8	16	64	80

Tabulate results for different numbers of frogs

Number of frogs		Minimum number of moves		
Red Frog	Green Frog	Slides	Jumps	Moves
1	1	2	1	3
1	2	3	2	5
1	3	4	3	7
1	4	5	4	9
1	5	6	5	11
1	6	7	6	13
2	1	3	2	5
2	2	4	4	8
2	3	5	6	11
2	4	6	8	14
2	5	7	10	17
2	6	8	12	20
3	1	4	3	7
3	2	5	6	11
3	3	6	9	15
3	4	7	12	19
3	5	8	15	23
3	6	9	18	27
4	1	5	4	9
4	2	6	8	14
4	3	7	12	19
4	4	8	16	24
4	5	9	20	29
4	6	10	24	34
5	1	6	5	11
5	2	7	10	17
5	3	8	15	23
5	4	9	20	29
5	5	10	25	35
5	6	11	30	41
6	1	7	6	13
6	2	8	12	20
6	3	9	18	27
6	4	10	24	34
6	5	11	30	41
6	6	12	36	48

Much of the data in the table has been duplicated. We can see from the corresponding colours that the number of moves for two red frogs and three green frogs is the same as the number of moves for three red frogs and two green frogs.

- **Look for patterns**

It would be easier to break the data down into sections.

Pattern for number of slides when equal number of frogs on each side

Red Frogs	Green Frogs	Slides
2	2	4
3	3	6
4	4	8
5	5	10
6	6	12
7	7	14
8	8	16

+2
+2
+2
+2
+2
+2
+2

The simplest pattern to recognise is that the number of slides increases by two each time.

Red Frogs	Green Frogs	Slides
2	+2	= 4
3	+3	= 6
4	+4	= 8
5	+5	=10
6	6	12
7	7	14
8	8	16
a	b	a+b

The number of slides can be found by adding the number of red frogs with the number of green frogs.

Red Frogs	Green Frogs	Jumps
2	2	4
3	3	9
4	4	16
5	5	25
6	6	36
7	7	49
8	8	64

The children may notice that the numbers increase by consecutive odd numbers.

Red Frogs	Green Frogs	Jumps
2	X 2	= 4
3	X 3	= 9
4	X 4	= 16
5	X 5	= 25
6	6	36
7	7	49
8	8	64
a	b	axb

They may also notice that all of the results are square numbers.

This should lead to realising that the number of jumps is equal to the number of red frogs multiplied by the number of green frogs.

The patterns that have been found by looking at the data accumulated for the same number of frogs on each side can make it easier to spot the patterns when we move on to looking at patterns when there are a different number of frogs on each side.

Again it will be easier to break the results down.

Pattern for 1 frog on one side

Number of Frogs		Number of Moves		
Red Frog	Green Frog	Slides	Jumps	Moves
1	1	2	1	3
1	2	3	2	5
1	3	4	3	7
1	4	5	4	9

Number of slides increases by 1 each time

Number of jumps increases by 1 each time

Number of moves increases by 2 each time (slides + jumps)

Pattern for 2 frogs on one side

Number of frogs		Number of moves		
Red Frog	Green Frog	Slides	Jumps	Moves
2	1	3	2	5
2	2	4	4	8
2	3	5	6	11
2	4	6	8	14
2	5	7	10	17
2	6	8	12	20

Number of slides increases by 1 each time.

Number of jumps increases by 2 each time.

Number of moves increases by 3 each time (slides + jumps)

Pattern for 3 frogs on one side

Number of frogs		Number of moves		
Red Frog	Green Frog	Slides	Jumps	Moves
3	1	4	3	7
3	2	5	6	11
3	3	6	9	15
3	4	7	12	19
3	5	8	15	23
3	6	9	18	27

Number of slides increases by 1 each time.

Number of jumps increases by 3 each time.

Number of moves increases by 4 each time (slides + jumps)

The children should deduce from this that the number of moves is equal to the number of slides added to the number of jumps.

Look at the relationship between the slides for varying numbers of red frogs

Red Frog	Green Frog	Slides
1	1	2
1	2	3

Each table of results increases by 1 each time.

They should notice that the last result for each of these tables is the same as the first result in the next table.

Red Frog	Green Frog	Slides
2	1	3
2	2	4

Discuss number bonds to 10. How many ways can we make the numbers that appear in the last column?

Red Frog	Green Frog	Slides
3	1	4
3	2	5

They should be able to deduce from these results that the number of slides is found by adding the number of red frogs to the number of green frogs.

Look at the relationship between the jumps for varying numbers of frogs

Red Frog	Green Frog	Jumps
1	1	1
1	2	2
1	3	3

They should notice that the increase in the number of jumps varies; table 1 by 1, table 2 by 2; table 3 by 3, table 4 by 4.

They should also notice that the first result in each table is the same as the number of frogs that have been kept constant for that set of results.

Red Frog	Green Frog	Jumps
2	1	2
2	2	4
2	3	6

They should notice that the second number in each set of results is double the first number and that the third number is 3 times larger than the first number.

Red Frog	Green Frog	Jumps
3	1	3
3	2	6
3	3	9

They should be able to deduce from this that the increase in the number of jumps increases in multiples of the number of frogs that is kept constant.

They should be able to find from this that the number of leaps is equal to the number of red frogs multiplied by the number of green frogs.

Red Frog	Green Frog	Jumps
4	X 1	= 4
4	X 2	= 8
4	X 3	= 12

Solutions

- To find the minimum number of slides add the number of red frogs to the number of green frogs.
- To find the minimum number of jumps multiply the number of red frogs by the number of green frogs.
- To find the minimum number of moves add the minimum number of slides to the minimum number of jumps.
- NB Children will most probably use specific values of n to explain their logic, not a generalised one.
- To find the generalised statement;

Let the number of red frogs be a
Let the number of green frogs be b

So if we need to add the number of red frogs to the number of green frogs to find the minimum number of slides we can write this as

$$(a + b)$$

If we need to multiply the number of red frogs by the number of green frogs to find the minimum number of jumps we can write this as

$$(a \times b)$$

And if we need to add both of these answers together we can write this as

$$(a + b) + (a \times b)$$

This is our general tem $(a + b) + (a \times b)$

- To check the generalisation works, substitute a for a number of red frogs and b for a number of green frogs

$$\text{Number of red frogs} = 5$$

$$\text{Number of green frogs} = 6$$

$$= (5 + 6) + (5 \times 6)$$

$$= 11 + 30$$

$$\text{Number of moves} = \mathbf{41}$$