The Logistic Map

x_{n+1}=$2x_n\ +1$ when $x_0=3$

x_0	
x_1	
x_2	
x_3	
x_4	

x_{n+1}=$0.5x_n$ when $x_0=100$

x_0	
x_1	
x_2	
x_3	
x_4	

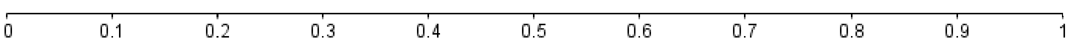
x_{n+1}=$0.5x_n\ +1$ when $x_0=3$

x_0	
x_1	
x_2	
x_3	
x_4	

Fill in the table for each sequence above. What is going to happen to each one if you carry on the sequence for a long time?

Now we are going to look at a special sequence used to analyse poplulation size of, say, rabbits.

The terms x_0 , x_1 , x_2 , x_3 , x_4 , x_5 … x_n are going to represent the population of rabbits every year. The numbers in the sequence are always going to be between 0 and 1, and will represent a proportion of rabbits



no rabbits

maximum rabbits

The sequence is called The Logistic Map and looks like this:

$$x_{n+1}=ax_n\,(1-x_n)$$

We are always going to start with $x_0=0.5$ (half capacity of rabbits), but we are going to try different values of a, where a represents the fertility of the rabbits.

a=2.3

x_0	
x_1	
x_2	
x_3	
x_4	

a=0.65

x_0	
x_1	
x_2	
x_3	
x_4	

a=3.2

x_0	
x_1	
x_2	
x_3	
x_4	

What is happening for each sequence?

Try creating a graph on Geogebra with y axis being the population (from 0 to 1) and the x axis being the number in the sequence (say from 0 to 50 to show terms x_0 , x_1 , x_2 , x_3 , x_4 , x_5 … x_{50}). The first point on your graph will be (0,0.5) as we are starting with a proportion of 0.5 rabbits. You can use a slider to represent a so that you can see how the sequence changes as you change a.

See the Geogebra instructions if you need some help.

You should see some very strange behaviour for different values of a.